Math 114 HW 8

due Wednesday, 6/22

1. Compute the line integral of the function $f(x, y, z) = xy + yz + x^2 + 2$ along the straight line path C that goes from the point $(1, 3, 4)$ to the point $(-1, 2, 1)$.
2. (This is the problem discussed in class, with f possibly changed.) Compute the line integral of the function $f(x, y, z) = x + y + z$ along the path C which goes first from the point $(1, 0, 0)$ to the point $(0, 1, 0)$ in a circular arc along $x^2 + y^2 = 1$ in the first quadrant of the x-y plane and then goes from $(0, 1, 0)$ to $(0, 0, 1)$ in a circular arc along $y^2 + z^2 = 1$ in the first quadrant of the y-z plane.
3. (Fall 2010) Evaluate $\int_C x^2 dx + y^2 dy + z^2 dz$ where C is the straight line segment from $(1, 2, 3)$ to $(2, 3, 4)$.
4. (Fall 2013) Find the work done by the force field

   $$\vec{F}(x, y) = e^y \sin(x)\hat{\mathbf{i}} - (e^y \cos(x) - \sqrt{1 + y})\hat{\mathbf{j}}$$

   in moving a particle from $(-\pi, \pi^2)$ to (π, π^2) along the parabola $y = x^2$.
5. (**Optional problem for extra credit**) (Spring 2014) Let $\vec{F}$ be the vector field

   $$\vec{F} = \langle y^2 + 2xe^y + 1, 2xy + x^2 e^y + 2y \rangle.$$

   Compute the work integral $\int_C \vec{F} \cdot d\vec{r}$ where C is the path

   $$\vec{r}(t) = \sin t\hat{\mathbf{i}} + \cos t\hat{\mathbf{j}}, \quad 0 \leq t \leq \frac{\pi}{2}$$

6. Compute the flux of the field $\vec{F} = 4y\hat{\mathbf{i}} + 3x\hat{\mathbf{j}}$ across the circle $\vec{r}(t) = \cos t\hat{\mathbf{i}} + \sin t\hat{\mathbf{j}}$.
7. Suppose $\vec{F}$ is the gradient field of the function $f(x, y, z) = x^2 + y^2 + z^2 + xyz$. Without actually computing an integral, find the work done by $\vec{F}$ in moving an object along the straight line from $(1, 1, 1)$ to $(2, 2, 3)$.
8. Which of the following fields are conservative?
   (a) $yz\hat{\mathbf{i}} + xz\hat{\mathbf{j}} + xy\hat{\mathbf{k}}$
   (b) $-y\hat{\mathbf{i}} + x\hat{\mathbf{j}}$
9. Find a potential function for the following fields :
   (a) $\vec{F} = 5x\hat{\mathbf{i}} + 2y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$
   (b) $\vec{F} = (y \sin z)\hat{\mathbf{i}} + (x \sin z)\hat{\mathbf{j}} + (xy \cos z)\hat{\mathbf{k}}$

10. Evaluate the integral

$$\int_C (y+z)dx + (x+z)dy + (x+y)dz$$

over any path C between the points $(1,1,1)$ and $(3,2,-1)$, by finding a suitable potential function.