Math 114 HW 9

due Monday, 6/27

1. (Spring 2011) Evaluate
$$\int_C xy^3dx + 3x^2y^2dy$$
where C is the boundary of the region in the first quadrant enclosed by the x-axis, the line $x = 1$ and the curve $y = x^3$, traversed counter-clockwise.

2. (Fall 2010) Find the value of the line integral $I = \displaystyle\int_C (x^2+y)dx+(y^2-x)dy$ where C is the triangle with vertices $(x, y) = (0, 0)$, $(3, 0)$, $(0, 4)$ traversed counterclockwise.

3. (Fall 2011) Evaluate the integral
$$\int_C \left(y + \sin\left(e^{x^2}\right)\right) dx - 2xdy,$$
where C is the circle $x^2 + y^2 = 1$ traversed counterclockwise.

4. Use Green's theorem to evaluate the integral
$$\oint_C (3ydx + 2xdy)$$
where C is the boundary of $0 \leq x \leq \pi$, $0 \leq y \leq \sin x$.

5. (Fall 2013) Use Green's theorem to evaluate the line integral $\displaystyle\int_C \vec{F} \cdot d\vec{r}$ where
$$\vec{F} = \left(e^{y^2} - 2y\right)\hat{\mathbf{i}} + \left(2xye^{y^2} + \sin(y^2)\right)\hat{\mathbf{j}}$$
and C goes along a straight line from $(0, 0)$ to $(1, 2)$ and continues along a straight line to $(3, 0)$.

6. (Fall 2003) Find the surface area of that portion of the paraboloid $z = 4 - x^2 - y^2$ that is above the plane $z = 1$.

7. Find the surface area of the portion of the paraboloid $y = x^2 + z^2$ that is between the planes $y = 1$ and $y = 4$.

8. The rectangular coordinate equation $z = \sqrt{x^2 + y^2}$ represents the cone $\phi = \frac{\pi}{4}$. Find the area of the surface cut from the hemisphere $x^2 + y^2 + z^2 = 2$, $z \geq 0$ by the cone $z = \sqrt{x^2 + y^2}$.

9. Let S be the part of the paraboloid $z = 17 - x^2 - y^2$ lying above the plane $z = 1$, oriented with normal vector pointing downward. Compute the flux of $\nabla \times \vec{F}$ across S, where $\vec{F}$ is the vector field $\vec{F} = \langle -yz, xz^2, xyz \rangle$.
Hint : Use Stokes' theorem.

10. Let C be the curve which is the boundary of the square $1 \leq x \leq 2$, $2 \leq y \leq 3$. Use the surface integral in Stokes' theorem to calculate the line integral of the vector field $\vec{F} = (y^2 + z^2)\hat{\mathbf{i}} + (x^2 + y^2)\hat{\mathbf{j}} + (x^2 + y^2)\hat{\mathbf{k}}$ over the curve C traversed counterclockwise when viewed from above.