Math 114 Practice Questions for Midterm 1

1. Find the radius of the sphere $3x^2 + 3y^2 + 3z^2 - 12x + 6y + 18z - 33 = 0$.

2. Let $\vec{(}u) = 3a\hat{\mathbf{i}} + 6\hat{\mathbf{j}} + 10\hat{\mathbf{k}}$ and let $\vec{v} = 14\hat{\mathbf{i}} + b\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$. Find values of a and b so that $\vec{u}$ and $\vec{v}$ are parallel.

3. Let $\vec{v}$ and $\vec{w}$ be two unit vectors and let θ be the angle between them. Find $\left|\frac{1}{2}\vec{v} - \vec{w}\right|$ in terms of θ.

   Hint : Use the dot product.

4. For a pair of vectors $\vec{u} = u_1\hat{\mathbf{i}} + u_2\hat{\mathbf{j}} + u_3\hat{\mathbf{k}}$ and $\vec{v} = v_1\hat{\mathbf{i}} + v_2\hat{\mathbf{j}} + v_3\hat{\mathbf{k}}$, find the components of the vector $\vec{u} \times \mathbf{proj}_{\vec{u}}\vec{v}$. Can you find the answer without explicitly computing out the cross product? Explain your answer.

5. Let L be the line that passes through the point $P(10, 2, 1)$, lies in the plane $2x + 5y - z = 29$, and is perpendicular to the line

x	=	7	-	2t
y	=	4	+	2t
z	=	5	+	6t.

   Find the distance of L from the point $S(5, 2, -1)$.

6. Find the distance between the planes $x - 2y + 2z = 7$ and $x - 2y + 2z = 23$.

7. Find the point on the line

x	=	2	-	2t
y	=	-2	+	1t
z	=	5	+	2t

   which is at a distance of 5 units from (2,-2,5).

8. For the curve given by

$$\vec{r}(t) = \cos t\hat{\mathbf{i}} + \tan t\hat{\mathbf{j}} + \frac{5}{\pi}t^2\hat{\mathbf{k}},$$

   find the parametric equations of the line tangent to the curve at the point $\left(\frac{1}{2}, \sqrt{3}, \frac{5\pi}{9}\right)$.

9. A particle moves with acceleration $\vec{a}(t) = 2t\hat{\mathbf{i}} + 3t^3\hat{\mathbf{j}} + \frac{2}{t^2}\hat{\mathbf{k}}$. If its velocity at time $t = 2$ was $2\hat{\mathbf{i}} + 5\hat{\mathbf{j}} - \hat{\mathbf{k}}$ and its position at time $t = 1$ was $\hat{\mathbf{i}} + 3\hat{\mathbf{k}}$, find the coordinates of the position of the particle at time $t = 2$.

10. A projectile is fired from the ground at a velocity of 40 m/s. What is the minimum launch angle so that the projectile can get to the other side of a 60 m high wall?

11. Find the unit tangent vector to the curve $\vec{r}(t) = \sin(t^2)\hat{\mathbf{i}} + \cos(t^2)\hat{\mathbf{j}} + t^2\hat{\mathbf{k}}$ at the point $(0, -1, \pi)$.

12. Find the length of the curve $\vec{r}(t) = e^{2t}\hat{\mathbf{i}} - e^{-2t}\hat{\mathbf{j}} + 2\sqrt{2}t\hat{\mathbf{k}}$ between the points $(1, 1, 0)$ and $(e^2, e^{-2}, 2\sqrt{2})$.

13. Find the curvature and unit normal vector of the curve $\vec{r}(t) = (3t-1)\hat{\mathbf{i}} + 2\cos 2t\hat{\mathbf{j}} + 2\sin 2t\hat{\mathbf{k}}$ at the point corresponding to $t = \frac{\pi}{8}$.

14. Which of the following limits exist?

    (a) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^4\sqrt{y} - 2x^2y + y^{\frac{3}{2}}}{\sqrt{y} - x^2}$

    (b) $\lim\limits_{(x,y)\to(0,0)} f(x,y)$, where f is the function defined by : $f(x,y) = \begin{cases} \frac{x^3+y^3}{xy} & \text{if } x \neq 0 \text{ and } y \neq 0 \\ 0 & \text{if } x = 0 \text{ or } y = 0 \end{cases}$