Math 114 Practice Questions for Midterm 2

1. For each of the following functions f, find f_x, f_y, f_{xx}, f_{xy}, f_{yy}.
   (a) $f(x,y) = e^{2x+3y+1}$
   (b) $f(x,y) = e^{\frac{y}{x}} + xy + y^2$
   (c) $f(x,y) = \ln(xy) + \cos(x^3 + 3xy) + \sqrt{x^2+y^2}$
2. If $z = cos(xy) + xy^2$, $x = u + v^2$, and $y = uv$, then find the value of $\frac{\partial z}{\partial u}$ when $u = 1$ and $v = 1$.
3. Find the directional derivative of the function $g(x,y,z) = x^2 + 3y^2 + 5z^2$ at the point $(1,2,-1)$ in the direction of the vector $\vec{v} = 2\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$.
4. Let $f(x,y) = x^2 + 5xy + y^3$. At the point $(-2,1)$, find the directions $\vec{u}$ (unit vectors) in which $D_{\vec{u}}f$ is largest and smallest.
5. In what directions is the directional derivative of $f(x,y) = \dfrac{1}{x^2+y^2}$ at point $(1,-1)$ equal to zero?
6. Find the equations of the tangent plane and normal line to the surface $x^2 + xy + y^2 - yz + z^2 = 13$ at the point $(2,1,3)$.
7. For the sphere of radius 2 centered at (1,3,-2), find the point(s) on it where the tangent plane is parallel to the plane $x + y = 5$.
8. For the function $f(x,y) = x^4 + y^4 + 4xy$, find all critical points and classify them as points of local maximum, minimum and saddle points.
9. Find and identify all critical points of $g(x,y) = \dfrac{1}{x} + xy + \dfrac{1}{y}$.
10. Find the absolute maximum and minimum of $f(x,y) = 3x - y + 7$ on the disk $x^2 + y^2 \leq 4$.
11. Find the maximum volume of a rectangular (solid) box of side lengths a, b and c under the constraint that $a + b^2 + c \leq 25$.

    For the chapter on Multiple Integrals it would be helpful to do the following problem numbers from the textbook.

12. Section 15.1 : 8, 16, 21
13. Section 15.2 : 37, 42, 45, 50, 52
14. Section 15.3 : 4, 8, 9, 11, 12

15. Section 15.4 : 11, 12, 17, 20
16. Section 15.5 : 15, 21, 26, 27
17. Section 15.7 : 14, 34, 38, 43
18. Section 15.8 : 1, 2, 6, 9, 10