# Math 114 Practice Questions for Midterm 2-Answers

1. (a) $f_x = 2e^{2x+3y+1}$,
$f_y = 3e^{2x+3y+1}$,
$f_{xx} = 4e^{2x+3y+1}$,
$f_{xy} = 6e^{2x+3y+1}$,
$f_{yy} = 9e^{2x+3y+1}$

   (b) $f_x = \dfrac{-y}{x}e^{\frac{y}{x}} + y$,
$f_y = \dfrac{1}{x}e^{\frac{y}{x}} + x + 2y$,
$f_{xx} = \dfrac{y^2}{x^4}e^{\frac{y}{x}} + \dfrac{2y}{x^3}e^{\frac{y}{x}}$,
$f_{xy} = 1 - \dfrac{1}{x^2}e^{\frac{y}{x}} - \dfrac{y}{x^3}e^{\frac{y}{x}}$,
$f_{yy} = \dfrac{1}{x^2}e^{\frac{y}{x}} + 2$

   (c) $f_x = \dfrac{1}{x} - (3x^2+3y)\sin(x^3+3xy) + \dfrac{x}{\sqrt{x^2+y^2}}$,
$f_y = \dfrac{1}{y} - 3x\sin(x^3+3xy) + \dfrac{y}{\sqrt{x^2+y^2}}$
$f_{xx} = \dfrac{-1}{x^2} - (3x^2+3y)^2\cos(x^3+3xy) - 6x\sin(x^3+3xy) + \dfrac{y^2-x^2}{(x^2+y^2)^{\frac{3}{2}}}$
$f_{xy} = -(9x^3+9xy)\cos(x^3+3xy) + 3\sin(x^3+3xy) - \dfrac{xy}{(x^2+y^2)^{\frac{3}{2}}}$
$f_{yy} = \dfrac{-1}{y^2} - 9x^2\cos(x^3+3xy) + \dfrac{x^2-y^2}{(x^2+y^2)^{\frac{3}{2}}}$

2. $5 - 3\sin(2)$

3. $-10$

4. $D_{\vec{u}}f$ is largest in the direction of the unit vector $\dfrac{\hat{\mathbf{i}}}{\sqrt{50}} - \dfrac{7\hat{\mathbf{j}}}{\sqrt{50}}$ and is smallest in the direction of the unit vector $\dfrac{-\hat{\mathbf{i}}}{\sqrt{50}} + \dfrac{7\hat{\mathbf{j}}}{\sqrt{50}}$

5. $\dfrac{\hat{\mathbf{i}}}{\sqrt{2}} + \dfrac{\hat{\mathbf{j}}}{\sqrt{2}}$ and $-\dfrac{\hat{\mathbf{i}}}{\sqrt{2}} - \dfrac{\hat{\mathbf{j}}}{\sqrt{2}}$

6. Tangent plane : $5x + y + 5z = 26$; normal line : $x = 2 + 5t$, $y = 1 + t$, $z = 3 + 5t$.

7. $\left(1+\sqrt{2}, 3+\sqrt{2}, -2\right)$ and $\left(1-\sqrt{2}, 3-\sqrt{2}, -2\right)$

8. $(0,0)$ : saddle point; $(1,-1)$ : point of local minimum; $(-1,1)$ : point of local minimum.

9. One critical point : (1,1), and it is a point of local minimum

10. $\left(3\sqrt{\frac{2}{5}}, -\sqrt{\frac{2}{5}}\right)$ is the point of absolute maximum, and the maximum value is $7+10\sqrt{\frac{2}{5}}$. The point $\left(-3\sqrt{\frac{2}{5}}, \sqrt{\frac{2}{5}}\right)$ is the point of absolute minimum, and the minimum value is $7-10\sqrt{\frac{2}{5}}$.

11. $100\sqrt{5}$