

Math 114 Quiz 1

Thu, 5/26

Name :

1. (a) Compute the cross product $\mathbf{u} \times \mathbf{v}$, where

$$\mathbf{u} = \hat{i} + 2\hat{j} + 3\hat{k}, \quad \mathbf{v} = 2\hat{i} + \hat{j} + \hat{k}$$

- (b) Use the result of part (a) to find the equation of the plane through the point $P(1, 5, 7)$, whose normal vector is perpendicular to both \mathbf{u} and \mathbf{v} .

Solution: (a) $\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & 1 & 1 \end{vmatrix}$

$$= \hat{i} \begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} - \hat{j} \begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} + \hat{k} \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix}$$

$$= (2-3)\hat{i} - (1-6)\hat{j} + (1-4)\hat{k}$$

$$= \boxed{-\hat{i} + 5\hat{j} - 3\hat{k}}$$

(b) $\vec{u} \times \vec{v}$ is a vector that is perpendicular to both \vec{u} and \vec{v} .
Therefore $\vec{u} \times \vec{v} = -\hat{i} + 5\hat{j} - 3\hat{k}$ is parallel to the normal vector of the plane.

Equation of the plane: $\boxed{-(x-1) + 5(y-5) - 3(z-7) = 0}$

i.e. $-x + 1 + 5y - 25 - 3z + 21 = 0$

i.e. $\boxed{x - 5y + 3z = -3}$

↑
(BOTH ARE CORRECT)