Math 114 Quiz 2

Tue, 5/31

Name :

1. Suppose the velocity of a particle is given by $\vec{v}(t) = (t^3 + 4)\hat{\mathbf{i}} + \sin t\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$.
   (a) Find the position vector $\vec{r}(t)$, given that the particle starts at the point $(0, 2, -1)$.
   (b) Also find the acceleration vector at time $t = \pi$.

Solution: (a) $\vec{r}(t) = \int \vec{v}(t) + \vec{C}$

$= \left(\frac{t^4}{4} + 4t\right)\hat{i} - \cos t\,\hat{j} + 3t\hat{k} + \vec{C}$

we're given that $\vec{r}(0) = 0\hat{i} + 2\hat{j} - \hat{k} = 2\hat{j} - \hat{k}$

$\vec{r}(0) = \left(\frac{0^4}{4} + 4\times 0\right)\hat{i} - \cos 0\,\hat{j} + 3\times 0\,\hat{k} + \vec{C}$

$= -\hat{j} + \vec{C}$

therefore $-\hat{j} + \vec{C} = 2\hat{j} - \hat{k}$

so $\vec{C} = 3\hat{j} - \hat{k}$

so $\vec{r}(t) = \left(\frac{t^4}{4} + 4t\right)\hat{i} - \cos t\,\hat{j} + 3t\hat{k} + 3\hat{j} - \hat{k}$

$= \boxed{\left(\frac{t^4}{4} + 4t\right)\hat{i} + (3 - \cos t)\hat{j} + (3t - 1)\hat{k}}$

(b) acceleration $\vec{a}(t) = \frac{d\vec{v}}{dt} = 3t^2\hat{i} + \cos t\,\hat{j} + 0\hat{k}$

$= 3t^2\hat{i} + \cos t\,\hat{j}$

$\vec{a}(\pi) = 3\pi^2\hat{i} + \cos\pi\,\hat{j} = \boxed{3\pi^2\hat{i} - \hat{j}}$