Math 114 Quiz 3

Thu, 6/2

Name :

1. Find the length of the curve given by

$$\vec{r}(t) = 2t^2\hat{\mathbf{i}} + 8t\hat{\mathbf{j}} + \frac{16}{3}t\sqrt{t}\hat{\mathbf{k}}$$

between the points $(8, 16, \frac{32\sqrt{2}}{3})$ and $(50, 40, \frac{80\sqrt{5}}{3})$.

Solution : $\vec{v}(t) = \frac{d\vec{r}}{dt} = 4t\,\hat{i} + 8\,\hat{j} + \frac{16}{3} \times \frac{3}{2} t^{1/2}\,\hat{k}$

$$= 4t\,\hat{i} + 8\,\hat{j} + 8t^{1/2}\,\hat{k}$$

so $|\vec{v}(t)| = \sqrt{(4t)^2 + 8^2 + (8t^{1/2})^2}$

$$= \sqrt{16t^2 + 64 + 64t}$$

$$= \sqrt{16(t^2 + 4 + 4t)} = \sqrt{16(t+2)^2}$$

$$= 4(t+2)$$

Also, the point $(8, 16, \frac{32\sqrt{2}}{3})$ corresponds to $2t_1^2 = 8$, $8t_1 = 16$, $\cancel{\frac{32\sqrt{2}}{3}}$ $\frac{16}{3}t_1\sqrt{t_1} = \frac{32\sqrt{2}}{3}$

i.e. $t_1 = 2$

and the point $\left(50, 40, \frac{80\sqrt{5}}{3}\right)$ corresponds to $2t_2^2 = 50$, $8t_2 = 40$, $\frac{16}{3}t_2\sqrt{t_2} = \frac{80\sqrt{5}}{3}$

i.e. $t_2 = 5$

Therefore, length of the curve is:

$$\int_2^5 |\vec{v}(t)|\,dt = \int_2^5 4(t+2)\,dt = 4\left(\frac{t^2}{2} + 2t\right)\Big|_2^5 = 2t^2 + 8t\Big|_2^5$$

$$= \cancel{4}(2 \times 5^2 + 8 \times 5) - (2 \times 2^2 + 8 \times 2) = (50 + 40) - (8 + 16)$$

$$= 90 - 24 = \boxed{66}$$