Math 114 Quiz 5

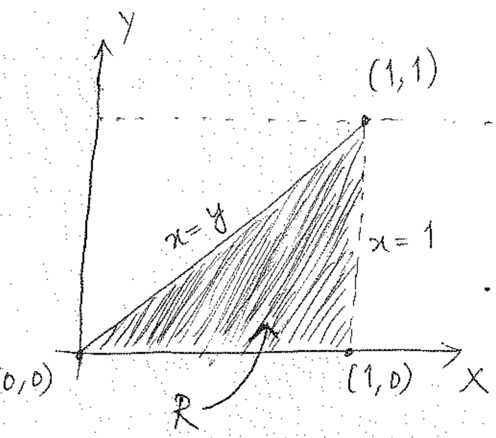
Tue, 6/14

Name :

1. Sketch the region of integration in the x-y plane and evaluate the integral :

$$\int_0^1 \int_y^1 x^4 e^{(x^2 y)} dx dy$$

Hint : You may want to reverse the order of integration.



Solution: reversing the order of integration,

$$\int_{x=0}^{1} \int_{y=0}^{x} x^4 e^{(x^2 y)} dy\, dx$$

$$= \int_{x=0}^{1} \left[x^4 \frac{e^{(x^2 y)}}{x^2} \right]_{y=0}^{x} dx$$

$$= \int_{x=0}^{1} \left[x^2 . e^{(x^2 y)} \right]_{y=0}^{x} dx = \int_{x=0}^{1} \left(x^2 e^{(x^2 \times x)} - x^2 e^{(x^2 \times 0)} \right) dx = \int_{x=0}^{1} \left(x^2 e^{x^3} - x^2 \right) dx$$

$$= \int_{x=0}^{1} x^2 e^{x^3} dx - \int_0^1 x^2 dx$$

First integral: $\int_0^1 x^2 e^{x^3} dx$. do u-sub, $u = x^3$, then $du = 3x^2 dx$, $\frac{du}{3} = x^2 dx$

and when $x=0$, $u = 0^3 = 0$

when $x = 1$, $u = 1^3 = 1$

$$= \int_0^1 e^u \frac{du}{3} = \frac{e^u}{3}\Big|_{u=0}^{1} = \frac{e^1 - e^0}{3} = \frac{e-1}{3}$$

Second integral: $\int_0^1 x^2 dx = \frac{x^3}{3}\Big|_0^1 = \frac{1^3}{3} - \frac{0^3}{3} = \frac{1}{3}$

$\therefore$ Final answer $= \frac{e-1}{3} - \frac{1}{3} = \boxed{\frac{e-2}{3}}$