

Math 114 Quiz 6

Thu, 6/23

Name :

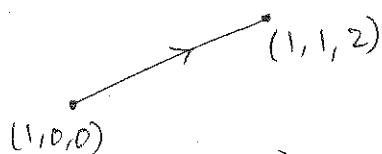
1. Find the work done by the force $\vec{F} = 4x\hat{i} + z\hat{j} + (z+3)\hat{k}$ in moving an object from the point $(1, 0, 0)$ to the point $(1, 1, 2)$ along a straight line.
2. Find a potential function for the conservative vector field

$$\vec{F} = (3y + yz^3)\hat{i} + (3x + xz^3)\hat{j} + 3xyz^2\hat{k}$$

Solution : 1.

Path: direction vector = $\langle 0, 1, 2 \rangle$

a parametrization for the path is:
 $\vec{r}(t) = 1\hat{i} + t\hat{j} + 2t\hat{k}, 0 \leq t \leq 1$



so, $\frac{d\vec{r}}{dt} = \hat{j} + 2\hat{k}$

$$\vec{F}(\vec{r}(t)) = 4\hat{i} + 2t\hat{j} + (2t+3)\hat{k}$$

$$\vec{F} \cdot \frac{d\vec{r}}{dt} = 2t + (4t+6) = 6t+6$$

$$\text{Work done} = \int_{t=0}^1 \vec{F} \cdot \frac{d\vec{r}}{dt} dt = \int_0^1 (6t+6) dt = 3t^2 + 6t \Big|_0^1 = \boxed{9}$$

2. $\vec{F} = \underbrace{(3y + yz^3)}_M \hat{i} + \underbrace{(3x + xz^3)}_N \hat{j} + \underbrace{3xyz^2}_P \hat{k}$

let $f(x, y, z)$ be the potential function.

$$\text{Then, } f = \int M dx = 3xy + xy z^3 + g(y, z)$$

$$f = \int N dy = 3xy + xy z^3 + h\left(\frac{x}{y}, z\right)$$

$$f = \int P dz = xy z^3 + k(x, y)$$

$$\Rightarrow \boxed{f(x, y, z) = 3xy + xy z^3 + C}$$