

MATH 201, QUIZ #3
Due Monday April 4. (beginning of your class)
Time: 45 minutes

1. (10 points) Sketch the region of integration and reverse the order of integration for the following:

$$(a) \int_{-\pi/2}^{\pi/2} \int_0^{\cos x} f dy dx = \int_0^1 \int_{-\cos^{-1} y}^{\cos^{-1} y} f dx dy$$

$$(b) \int_0^1 \int_{-x}^{x^2} f dy dx = \int_{-1}^0 \int_{-y}^1 f dx dy + \int_0^1 \int_{\sqrt{y}}^1 f dx dy$$

$$(c) \int_{-1}^2 \int_{x^2-1}^{x+1} f dy dx = \int_{-1}^0 \int_{-\sqrt{y+1}}^{\sqrt{y+1}} f dx dy + \int_0^3 \int_{y-1}^{\sqrt{y+1}} f dx dy$$

2. (10 points) Let W be the part of the first octant that lies below $3x + 2y + z = 6$. Set up limits for the triple integral

$$\int \int \int_W f dx dy dz = \int_0^6 \int_0^{\frac{6-z}{2}} \int_0^{\frac{6-z-2y}{3}} f dx dy dz$$

3. (10 points) Let R be the region of the plane where $x^2 + (y-1)^2 \leq 1$. Use polar coordinates to compute

$$\int \int_R x^2 + y^2 dA.$$

R is a disc, centered at $(0, 1)$. In polar coordinates, its equation is $r = 2 \sin \theta$.

$$\int \int_R x^2 + y^2 dA = \int_0^\pi \int_0^{2 \sin \theta} r^3 dr d\theta = \int_0^\pi \frac{r^4}{4} \Big|_0^{2 \sin \theta} d\theta = 4 \int_0^\pi \sin^4 \theta d\theta.$$

We use the double angle formulas $\sin^2 \theta = \frac{1 - \cos(2\theta)}{2}$ and $\cos^2 \theta = \frac{1 + \cos(2\theta)}{2}$

$$\begin{aligned} 4 \int_0^\pi \sin^4 \theta d\theta &= \int_0^\pi 1 - 2 \cos(2\theta) + \cos^2(2\theta) d\theta = \int_0^\pi 1 - 2 \cos(2\theta) + \frac{1 + \cos(4\theta)}{2} d\theta = \\ &= \left(\theta - \sin(2\theta) + \frac{1}{2}\theta + \frac{\sin(4\theta)}{8} \right) \Big|_0^\pi = \frac{3}{2}\pi. \end{aligned}$$