# Final Exam		Math 170		May 8, 2012

Name:

Section No.:

Instructions

- To recieve partial credit on multiple choice questions, you must provide justification for your answers.
- On short-answer questions, you must justify all steps.
- Four-function and scientific calculators only. No graphing or programmable calculators. No cell phones, laptops, or computers.
- One sheet of 8.5×11 paper is allowed for notes. Books and other forms of notes are not allowed.

	Out Of	Score
Part I	55	
Part II	50	
Part III	95	
Overall Score	200	

Part I (55 points)

Question 1. (5 points) Which fundamental mathematical concept or principle *fails* to be applicable when considering collections that are not finite?

A. Cardinality

B. the Pigeonhole Principle

C. One-to-one correspondance

D. Denumerability

Question 2. (10 points) Given an integers m and n, the *division algorithm* states that unique integers q and r can be found so that $0 \leq r < n$ and $m = nq + r$. In the case that $m = -13$ and $n = 3$, what must be the values of q and r?

A. $q = -4$, $r = -1$

B. $q = -\frac{13}{3}$, $r = 0$

C. $q = 4$, $r = 1$

D. $q = -5$, $r = 2$

Question 3. (10 points) Using a 24 hour clock system and assuming it is currently $13:00$ (which is 1 pm in a 12-hour clock system), what time will it be in one hundred hours?

A. $17:00$

B. $13:00$

C. $23:00$

D. $9:00$

Question 4. (10 points) In class, we used a certain method to determine that the value of

$$\phi \;=\; 1 + \cfrac{1}{1 + \cfrac{1}{1+\cfrac{1}{\ddots}}}$$

was precisely $\phi = \frac{1+\sqrt{5}}{2}$, or the golden ratio. Using this method, determine the value of

$$\rho \;=\; 2 + \cfrac{3}{2 + \cfrac{3}{2+\cfrac{3}{\ddots}}}$$

What is the value of ρ?

A. $\rho \;=\; \frac{1+\sqrt{5}}{2}$

B. $\rho \;=\; \frac{2+\sqrt{3}}{2}$

C. $\rho = 2.1$

D. $\rho \;=\; 2$

E. $\rho \;=\; 3$

F. $\rho = \frac{1+\sqrt{8}}{2}$

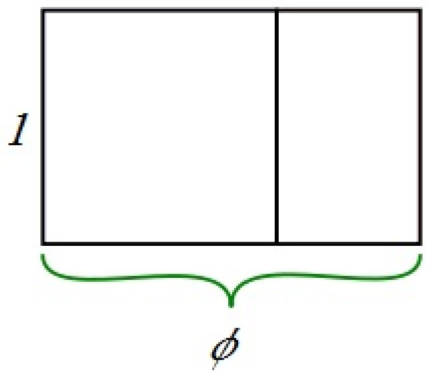
Question 5. (20 points) A genius classmate of yours tells you that he has devised a secret, super-smart method of listing *all* the real numbers between 0 and 1. Using his method, the first few numbers on his list are

$$\begin{array}{ll} 1. & 0.123123123\ldots \\ 2. & 0.141592654\ldots \\ 3. & 0.414213562\ldots \\ 4. & 0.199999099\ldots \\ 5. & 0.101001000\ldots \\ \vdots & \qquad \vdots \end{array}$$

Explain how you know with certainty that, no matter how smart he thinks his method is, you can find a number between 0 and 1 that is not on the list. Explain clearly both how to find the number, and how you know for sure that it is not on your genius friend's list.

Part II (50 points)

Question 6. (10 points) Any golden rectangle can be subdivided into a square and a second golden rectangle. Assume a rectangle of height 1 and length $\phi = \frac{1+\sqrt{5}}{2}$ is so divided, as indicated:



What is the area of the smaller golden rectangle?

A. $\frac{2}{1+\sqrt{5}}$

B. $\left(\frac{1+\sqrt{5}}{2}\right)^2$

C. $\left(\frac{2}{1+\sqrt{5}}\right)^2$

D. 1

E. $\frac{1+\sqrt{5}}{2}$

F. $\frac{\sqrt{5}-1}{2}$

Question 7. (5 points) A once-punctured sphere is topologically equivalent to

A. a sphere

B. a Klein bottle

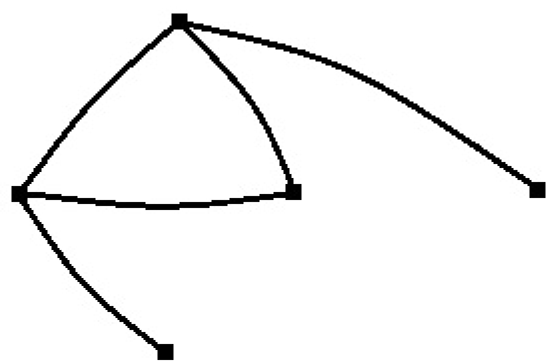
C. a cylinder

D. an annulus

E. a point

F. a disk

Question 8. (5 points) The graph depicted below has no Euler circuit.



What is the *fewest* number of edges one could add to the graph so that the result has an Euler circuit?

A. 1

B. 2

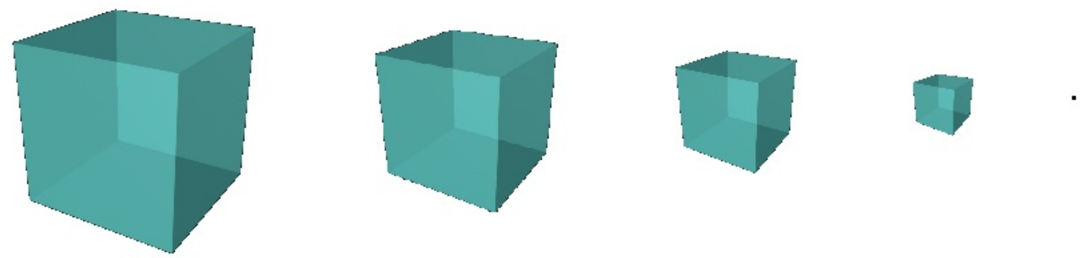
C. 3

D. 4

E. 5

F. 6

Question 9. (10 points) An object in 4-dimensional space passes through our ordinary 3-dimensional space, and we see the following sequence of shapes:



What 4-dimensional object has just passed through our space?

A) A tesseract (also known as a hypercube)

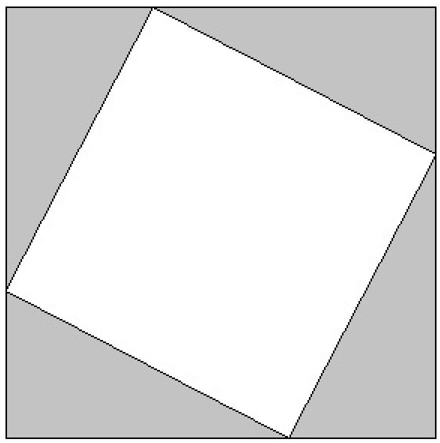
B) A hyper-tetrahedron

C) A hyper-pyramid with a cubic base

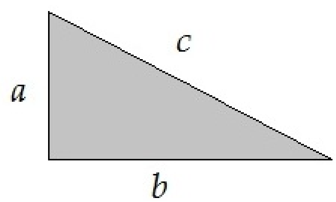
D) A hexacosichoron (a regular 4-dimensional polyhedron with 600 tetrahedral solid faces, 1200 triangular planar faces, 720 edges, and 120 vertices)

E) A hypersphere

Question 10. Consider the following figure



which consists of four triangles of the type



arranged so as to make a large square, with a smaller square in its interior.

Part a) (5 points) Determine the areas of the two squares that appear in the figure.

Part b) (5 points) Determine the areas of the four triangles that appear in the figure.

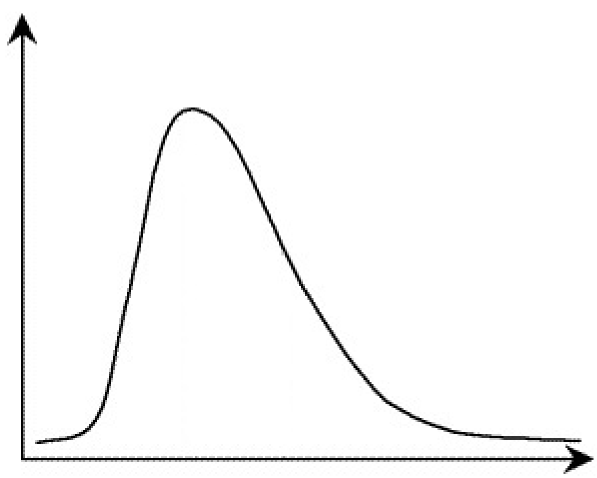
Part c) (10 points) Using your results from parts (a) and (b), prove the Pythagorean Theorem.

Part III (95 points)

Question 11. (10 points) If four fair dice are rolled at once, what are the odds of getting at least a pair?

A. 1
B. $\frac{1}{2}$
C. $\frac{5}{18}$
D. $\frac{17}{24}$
E. $\frac{2}{6}$
F. $\frac{13}{18}$

Question 12. (5 points) Given the following skewed distribution:



which statement best describes the relationship between the mean, median, and mode?

A. Median < Mode < Mean
B. Mean < Median < Mode
C. Mode < Mean < Median
D. Mode < Median < Mean

Question 13. (5 points) Assuming you have a loan with a 10% nominal interest rate and semi-annual compounding, what is your APR?

A. 9.75% B. 10% C. 10.25% D. 10.5%

Question 14. (15 points) Assume you take out a $\$1,000$ loan at a 10% rate of interest, compounded semi-annually. The terms of the loan stipulate that you pay the money back in two payments. The first payment of $400 is due after six months, and the remaining balance is due after another six months. What is the amount of your second payment?

A. $600 B. $682.85 C. $1000

D. $705.27 E. $50 F. $650

Question 15. (15 points) A few Nile Perch are introduced into Lake Victoria (the world's third largest fresh water lake), where they have abundant food supplies and no natural predators. In 10 years, it is estimated that the population has grown to 577, and in 11 years to 866. About how many Nile Perch will be in Lake Victoria in year 12?

A. 912	B. 2058	C. 860
D. 0	E. 1299	F. 100

Recall that the Verhulst model states $\frac{P_{n+1}-P_n}{P_n} = c(1 - P_n)$ where P_n is population density at year n, and the exponential growth model states $\frac{P_{n+1}-P_n}{P_n} = c$ where P_n is population (not population density) at year n.

Question 16. A particular fatal disease is present in 0.1% of the population, leaving 99.9% of the population without that disease. A new test for the disease becomes available, and the federal government decides to provide 100 million citizens free access to the test, at a cost to the government of \$20 per test. Assume the test is 99% accurate.

Part a) (15 points) Out of the 100 million people who took the test, determine how many false positives were produced, and how many false negatives were produced.

Part b) (10 points) Of those who recieved a positive test result (whether false or true), what proportion actually have the disease?

Part c) (15 points) Assume the government provides treatment to all those who tested positive, at a cost of \$100,000 per person. In this hypothetical situation, what is the cost to the government per life saved? (In your analysis you must consider the cost of the tests as well as the treatments, and also the fact that many positives were false positives—obviously the people with false positives were not in fact saved by the treatment, as their lives were never threatened by the disease in the first place.)

Part d) (5 points) Would you recommend the government fund this project? Why or why not?

Scratch.

Scratch.

Scratch.