Exam I

Math 170

2/15/2012

Name

Question	Out Of	Score
1	10	
2	5	
3	10	
4	10	
5	10	
6	10	
7	10	
8	5	
9	10	
10	10	
11	10	
Overall Score	100	

Part I (65 pts.)

Question 1. (10 pts.) On the left certain sets are listed, and on the right are different possible cardinalities. Match each set with a cardinality.

a. The set of US senators	A. Finite, but unknown
b. The set of natural numbers	B. Uncountable
c. The set of real numbers between 0 and 1	C. 100
d. The set of all books ever written	D. Countable
e. The set of all rational numbers	E. 435
f. The set of all atoms in the solar system	F. 8
g. The set of all real numbers	

Question 2. (5 pts.) The Lucas sequence L_1, L_2, L_3 ... is defined by setting $L_1 = 2$ and $L_2 = 1$, then recursively defining $L_n = L_{n-1} + L_{n-2}$ (as in the Fibonacci sequence) for $n > 2$. What is the 8th number in the Lucas sequence?

A. 1

B. 21

C. 47

D. 29

E. ∞

Question 3. (10 pts.) If L_n is the n^{th} number in the Lucas sequence and n is bigger than 1, which formula below correctly describes the sum $L_{n+1} + L_{n-1}$? (You are only asked to determine the most likely pattern, not to prove why the pattern holds).

A. $2L_n$

B. $5L_n$

C. $(L_n)^2 - (L_{n-1})^2$

D. $5F_{n-1}$

E. $F_n + L_n$

Note: The sequence $F_1, F_2, \ldots, F_n, \ldots$ is the Fibonacci sequence.

Question 4. (10 pts.) The *diagonal trick* can be used in many cases to find a number that is not part of a list. Consider the following list of 5 five-digit decimal numbers:

1. 0.12542
2. 0.99233
3. 0.67125
4. 0.19188
5. 0.64668.

Using the diagonal trick on this list, which number would you come up with?

A. 0.11111 B. 0.19188 C. 0.64668 D. 0.20299

Question 5. (10 pts.) If the time is currently 1 o'clock, what time will it be in

$$13 \times 11 \ + \ 6 \times 14 \times 13 \ + \ 5462 \times 12 \times 65536 \times 267^{625}$$

many hours?

A. 6 o'clock B. 5 o'clock C. 12 o'clock D. 1 o'clock E. A quarter to never

Question 6. (10 pts.) The following chart, if continued indefinitely, contains all positive rational numbers. Continuing the pattern indicated in the chart, a **one-to-one** correspondence between the positive rational numbers and the natural numbers can be established.

$$
\begin{tikzcd}
 & & \vdots & & & \\
5/1 & 5/2 & 5/3 & 5/4 & 5/5 & \\
4/1 & 4/2 & 4/3 & 4/4 & 4/5 & \\
3/1 \arrow[r, dash] \arrow[d, dash] & 3/2 \arrow[r, dash] & 3/3 \arrow[d, dash] & 3/4 & 3/5 & \cdots \\
2/1 \arrow[r, dash] & 2/2 \arrow[d, dash] & 2/3 \arrow[d, dash] & 2/4 & 2/5 & \\
1/1 \arrow[r, dash] & 1/2 & 1/3 \arrow[r, dash] & 1/4 & 1/5 &
\end{tikzcd}
$$

Under this correspondence, what rational number correpsonds to the natural number 13? (Don't forget about how to deal with repeats.)

A. 4/5 B. 1 C. 3/4 D. 5/2 E. 5

Question 7. (10 pts.) In the following partial UPC code, what is the correct check digit?

$$1 \quad 3 \quad 0 \quad 3 \quad 1 \quad 0 \qquad 9 \quad 5 \quad 0 \quad 0 \quad 6 \quad \Box \tag{1}$$

A. 5 B. 6 C. 7 D. 8 E. 9

Part II (35 pts.)

In this section of the exam you are asked to prove that $\sqrt{6}$ is irrational, by completing a series of steps.

Do not write down an uninterrupted proof; you will not recieve credit for doing so. Rather, answer the questions below individually.

To begin the proof, we note that if $\sqrt{6}$ were rational, we could write

$$\sqrt{6} = \frac{a}{b}, \tag{2}$$

where a and b are natural numbers. Since a fraction can always be expressed in lowest terms, we can assume from the outset that a and b have no common factors.

Question 8. (5 pts.) Show that both 2 and 3 appear in the prime decomposition of the number a^2.

Question 9. (10 pts.) Prove that both 2 and 3 appear in the prime decomposition of the number a.

Question 10. (10 pts.) Prove that both 2 and 3 appear in the prime decomposition of the number b^2, and therefore also in the prime decomposition of the number b.

Question 11. (10 pts.) To conclude, state why your arguments from Questions 8, 9, and 10 make it impossible for $\sqrt{6}$ to be a rational number.

Scratch.