Exam II   Math 170   March 28, 2012

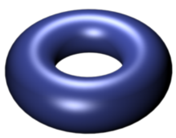
Name:

Section:

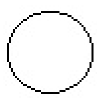
Question	Out Of	Score
1	10	
2	10	
3	10	
4	5	
5	10	
6	10	
7	10	
8	15	
9	5	
10	15	
Overall Score	100	

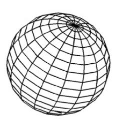
Part I (65 pts.)

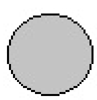
Question 1. (10 pts.) Below are two lists of geometrical objects. Match each object on the left with all topologically equivalent objects on the right (you do not have to justify your choices, but to recieve partial credit, you may).

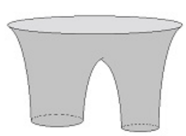
a.  (a torus)

b. 

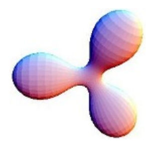
c.  (a circle)

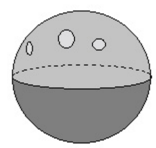
d.  (a sphere, hollow)

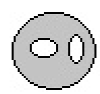
e.  (a disk)

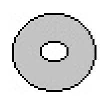
f.  (a "pair of pants")

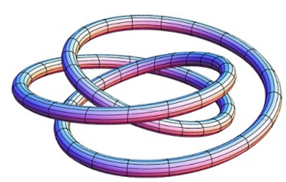
g. θ (the greek letter theta)

A.  (hollow)

B.  (a thrice-punctured sphere)

C.  (a twice-punctured disk)

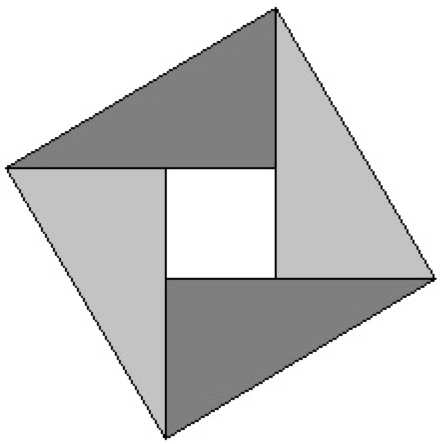
D.  (a once-punctured disk)

E.  (a knotted torus)

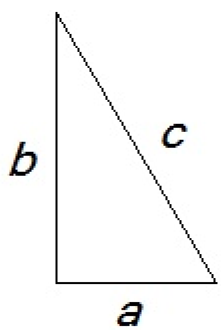
F. B (the letter "B")

G. D (the letter "D")

Question 2. (10 pts.) Consider the following diagram, used in a proof of the Pythagorean theorem



Specifically, this diagram consists of four right trianngles



arranged to make a larger square with a smaller square-shaped "hole." What is the area of the square-shaped "hole"?

A. $4ab$

B. $(b-a)^2$

C. c^2

D. $c^2 - a^2 - b^2$

E. 1

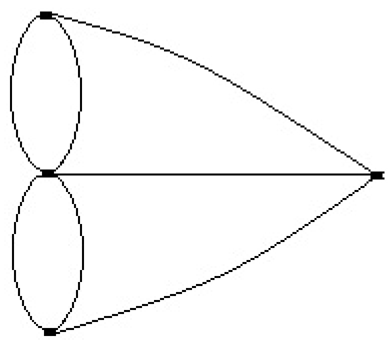
F. ∞

Question 3. (10 pts.) Below is a list of four ostensibly possible constructions; however some of them are actually impossible. Which construction(s) are impossible?

I. Construction of a graph with 8 vertices, 16 edges, and 11 regions

II. Constructions of a regular polyhedron with 12 faces

III. Construction of an Euler circuit in the following graph:



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A. I, II, and III are all impossible.

B. Only II and III are impossible.

C. Only I and II are impossible.

D. Only I is impossible.

E. Only II is impossible.

F. Only I and III are impossible.

G. All objects are possible.

Question 4. (5 pts.) Which Platonic solid is the dual of the cube?

A. The icosahdron

B. The cube (ie. the cube is self-dual)

C. The tetrahedron

D. The dodecahedron

E. The octahedron

Question 5. (10 pts.) The Verhulst population growth model is given by

$$\frac{P_{n+1} - P_n}{P_n} = k(1 - P_n) \tag{1}$$

where the number P_n is the population density at year n; that is, P_n is the ratio of population current population at year n to the maximum sustainable population.

Assume that $k = 0.1$ and $P_0 = 0.9$. What is the population density at year 2?

A. 1.09

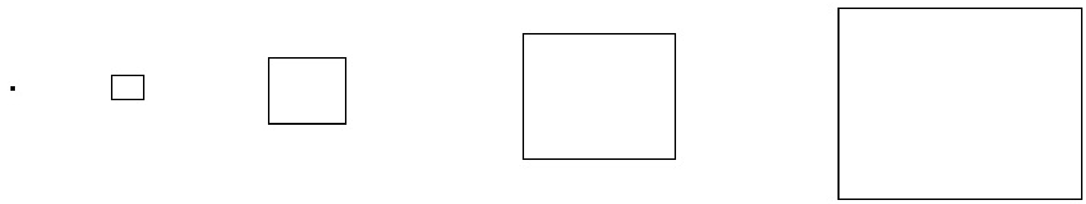
B. 1

C. .850909

D. 0.909909

E. 0.9

Question 6. (10 pts.) The following sequence of images represents a flatlander's view of a 3-dimensional object passing through 2-dimensional space.



What 3-dimensional object is it?

A. a sphere

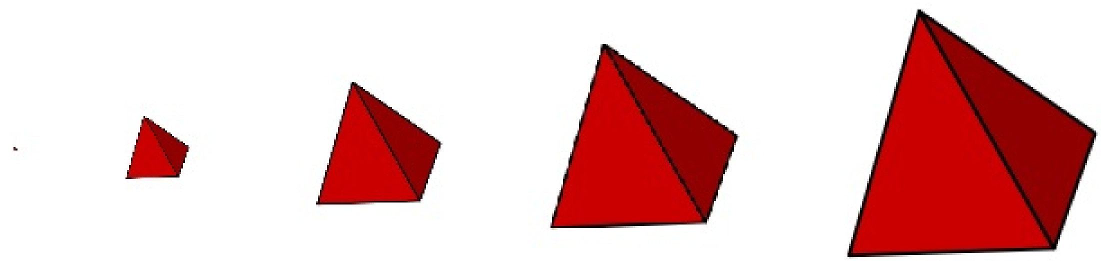
B. a cube

C. a pyramid with a square base

D. a triangular prism

E. a tetrahedron

Question 7. (10 pts.) The following sequence of images represents a 3-dimensional view of a 4-dimensional object passing through space.



What 4-dimensional object is it?

A. a hypersphere (ie. a sphere in 4-D space)

B. a tesseract, or hypercube (ie. a cube in 4-D space)

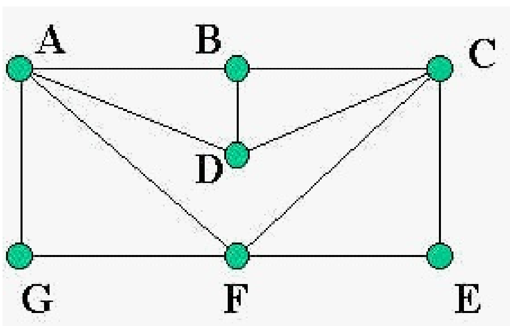
C. a hyper-pyramid with a cubic base (ie. a 4-D pyramidal object with a cubic base and square-based pyramidal sides)

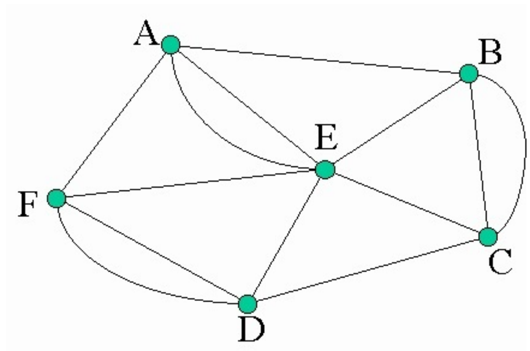
D. a tetrahedral prism (ie. a prism in 4-D space formed by extending a tetrahedron along a line segment)

E. a hyper-tetrahedron (ie. a 4-D pyramidal object with tetrahedral base and tetrahedral sides)

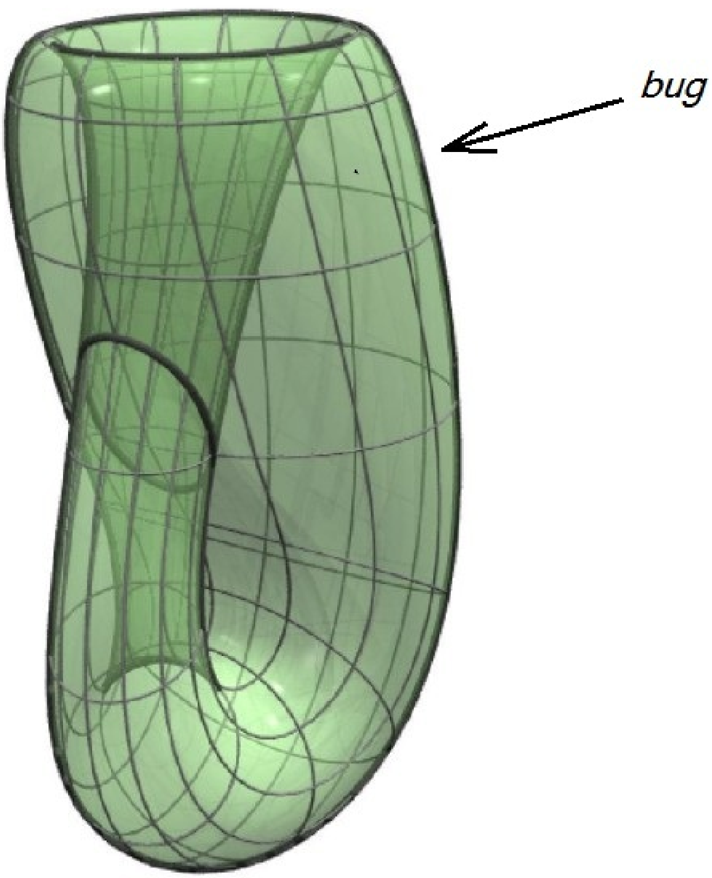
Part II (35 pts.)

Question 8. (15 points) Below are two graphs, which may or may not have Euler circuits. Determine if either has an Euler circuit, and if one of them does, describe it by writing down an ordered list of vertices visited.

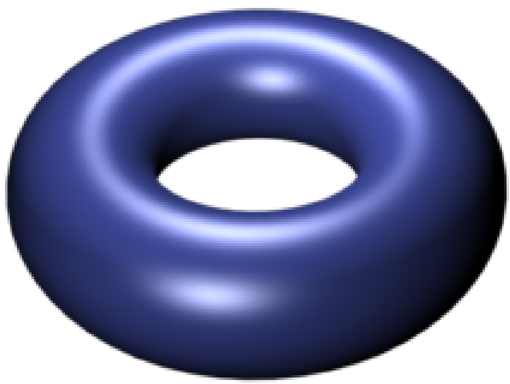


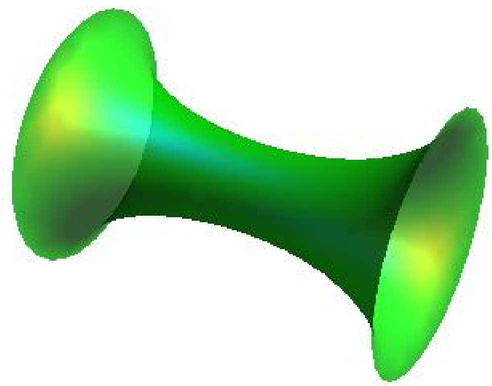


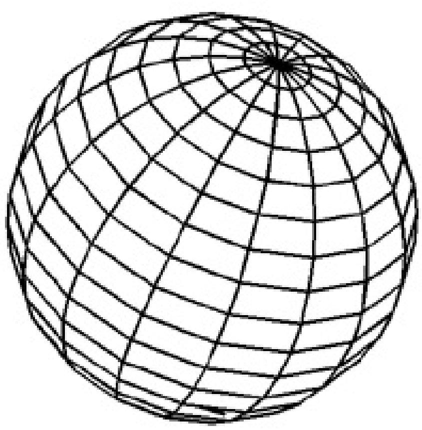
Question 9. (5 pts.) A bug is caught in a Klein bottle, as depicted. Describe how the bug can get out. You may use a drawing to suppliment your description, but you must also decribe the bug's journey in words. Be as spectific as possible.



Question 10. (15 pts.) Below are three shapes from the first question. First, detemine which of them, if any, are topologically equivalent (you don't have to justify this answer, but you may do so for partial credit). Second, pick two topologically **non**-equivalent shapes, and describe the topological changes needed to convert one of the objects into the other.







Scratch.