MATH 602 HOMEWORK 6, FALL 2014

1. Let F be a locally compact field. Let $\chi : F^\times \to \mathbb{C}^\times$ be a continuous character of $F^\times$.

   (a) Show that there exists a unique pair (σ, α) with $\sigma \in \mathbb{R}$ and $\alpha : F^\times \to \mathbb{C}_1^\times$ a unitary character such that $\chi = \alpha \cdot \omega_\sigma$. (We say that σ is the *real part* of χ)

   (b) Suppose that F is non-archimedian. Show that $\alpha(\mathscr{O}_F^\times)$ is finite.

   (c) Suppose that F is non-archimedian. Show that χ has finite order if and only if there exists a uniformizer π of F such that $\chi(\pi)$ is a root of unity.

2. Determine explicitly

   (a) the structure of the unitary dual of $\mathbb{Q}_p{}^\times$ as a topologial group, and

   (b) the structure of $\mathrm{Hom}_{\mathrm{cts}}(\mathbb{Q}_p{}^\times, \mathbb{C}^\times$ as a one-dimensional complex Lie group.

3. Compute explicitly the local L-function $L(\chi)$ and local constants $\varepsilon(\chi, \psi, dx)$ for all character χ of $\mathbb{Q}_p{}^\times$, all non-trivial unitary additive character ψ of $\mathbb{Q}_p$ and all Haar measure dx of $\mathbb{Q}_p$.

4. Let K be a number field and let $\chi : \mathbb{A}_K^\times \to \mathbb{C}^\times$ be an idele class character, i.e. χ is trivial on $K^\times$. Suppose that the image of $(K \otimes_{\mathbb{Q}} \mathbb{R})^\times$ under χ is a finite subgroup of $\mathbb{C}^\times$. Is χ a character of finite order? Either give a proof or give a counter-example.