Math 104 **Review**

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1. Find the volume of the following surfaces of revolution:
   (a) region bounded by $y = e^x$, $x = 0$, $x = 2$ and $y = 0$, rotated about the x-axis
   (b) region bounded by $y = e^x$, $x = 0$, $x = 2$ and $y = 0$, rotated about the y-axis
   (c) region bounded by $y = \sec(x)$, $y = 0$, $x = -\frac{\pi}{4}$ and $x = \frac{\pi}{4}$, rotated about $y = 2$
   (d) region bounded by $y = \sec(x)$, $y = 0$, $x = -\frac{\pi}{4}$ and $x = \frac{\pi}{4}$, rotated about the x-axis
   (e) region bounded by $y = e^x - 1$, $y = 2 - x$ and the x-axis, rotated about the x-axis
   (f) region bounded by $y = e^x - 1$, $y = 2 - x$ and the x-axis, rotated about the y-axis
   (g) region bounded by the x-axis, $y = 3x^4$, $x = 1$ and $x = -1$, rotated about the x-axis
   (h) region bounded by the x-axis, $y = 3x^4$, $x = 1$ and $x = -1$, rotated about the y-axis
   (i) region bounded by the x-axis, $y = 3x^4$, $x = 1$ and $x = -1$, rotated about the line $y = 3$
   (j) region in the first quadrant, bounded by the coordinate axes, $y = e^{-x}$, and the line $x = 1$, rotated about $x = 1$
   (k) region in the first quadrant, bounded by the coordinate axes, $y = e^{-x}$, and the line $x = 1$, rotated about the y-axis
2. Find the surface area of the following surfaces of revolution:
   (a) the curve $y = \sqrt{1 - x^2}$, $0 \leq x \leq \frac{1}{2}$, rotated about the x-axis
   (b) the curve $y = \frac{x^3}{3}$ $0 \leq x \leq 1$, rotated about the x-axis
   (c) the curve $9x = y^2 + 18$, $2 \leq x \leq 6$, rotated about the x-axis
   (d) the curve $y = \frac{x^3}{6} + \frac{1}{2x}$, rotated about the x-axis
3. Find the arc length of the following curves
   (a) $y = \ln(\sec(x))$, $0 \leq x \leq \frac{\pi}{4}$
   (b) $y = \frac{1}{4}(e^{2x} + e^{-2x})$, $0 \leq x \leq 1$
   (c) $y = \frac{1}{6}x^3 + \frac{1}{2x}$, $1 \leq x \leq 3$
   (d) $x = \frac{2}{3}y^{\frac{3}{2}}$, $0 \leq y \leq x$
4. Find the center of mass of the following regions
   (a) region bounded by $y = \arcsin(x), y = 0, x = \frac{1}{2}$ with constant density
   (b) region bounded by $y = x + \cos(x)$ and $y = 0$ for $0 \leq x \leq 2\pi$ with constant density
   (c) region bounded by the x-axis and $y = \frac{2}{x^2}$, $1 \leq x \leq 2$, with density $\delta(x) = x^2$

5. Evaluate each of the following integrals, or specify if it diverges:

   (a) $\displaystyle\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx$

   (b) $\displaystyle\int \frac{1}{x^3-1}\,dx$

   (c) $\displaystyle\int \frac{x}{\sqrt{4-x^2}}\,dx$

   (d) $\displaystyle\int_0^{\pi} 2x\cos x\,dx$

   (e) $\displaystyle\int \sin x \sin 2x\,dx$

   (f) $\displaystyle\int \cos^3 x \sin^4 x\,dx$

   (g) $\displaystyle\int (\ln x)^2\,dx$

   (h) $\displaystyle\int_0^2 \frac{x}{x^2-1}\,dx$

   (i) $\displaystyle\int \frac{\sin x\cos x}{\sin^2 x - 4}\,dx$

   (j) $\displaystyle\int x\sqrt{x^2+8}\,dx$

   (k) $\displaystyle\int_0^1 x^4 e^x\,dx$

   (l) $\displaystyle\int_0^1 \frac{1}{\sqrt{1-x}}\,dx$

   (m) $\displaystyle\int \frac{x^2}{x^4-1}\,dx$

   (n) $\displaystyle\int e^x \sin 4x\,dx$

   (o) $\displaystyle\int_0^{\frac{\pi}{2}} \cos^2 x \sin x\,dx$

6. Do the following integrals converge or diverge?

   (a) $\displaystyle\int_0^{\infty} \frac{\sin^2 x}{1+e^x}\,dx$

   (b) $\displaystyle\int_2^{\infty} \frac{x^2 e^x}{\ln x}\,dx$

   (c) $\displaystyle\int_1^{\infty} e^{x^2+x+1}\,dx$

   (d) $\displaystyle\int_1^{\infty} \frac{x^2-2}{x^4+3}\,dx$

7. Determine the constant c so that the following functions are probability density functions. Then, compute the mean and the median.

   (a) $p(x) = c(2 - x)$, $x \in [0, 4]$

   (b) $p(x) = \dfrac{c}{x^2 + 1}$, $x \in (-\infty, \infty)$

   (c) $f(x) = c \sin^2 x$, $x \in [0, \Pi]$

8. Find the solution of the following differential equations:

   (a) $y' = 1 - \frac{y}{x}$, $y(2) = -1$

   (b) $(1 + x)y' + y = \sqrt{x}$, $y(0) = 1$

   (c) $y' + 2y = 3$

   (d) $y' = \dfrac{\tan y}{x}$, $y(1) = \frac{\pi}{2}$

   (e) $e^{2x}y' + e^{2x}y = 2x$

   (f) $xy' + 3y = \dfrac{\sin x}{x^2}$

9. Do the following sequences converge or diverge? Compute the limits of the convergent sequences.

   (a) $\left\{(n^2 + n)^{\frac{1}{n}}\right\}$

   (b) $\left\{\arctan(\dfrac{n^2}{1 + n^2})\right\}$

   (c) $\left\{\cos(\dfrac{\sqrt{n}}{1 + n})\right\}$

   (d) $\left\{n^2(1 - \cos(\dfrac{1}{n}))\right\}$

   (e) $\{\sin(\arctan(\ln(n)))\}$

10. For each of the following series, say whether it converges (absolutely or conditionally) or diverges, and explain why.

    (a) $\displaystyle\sum_{n=1}^{\infty} \frac{e^n}{n!}$

    (b) $\displaystyle\sum_{n=1}^{\infty} n^3 e^{-n^2}$

    (c) $\displaystyle\sum_{n=1}^{\infty} \frac{4 + n}{3 + 2n}$

(d) $\sum_{n=1}^{\infty} \frac{n^2+5}{\sqrt{n^5+2}}$

(e) $\sum_{n=1}^{\infty} \frac{e^n}{(2+\frac{1}{n})^n}$

(f) $\sum_{n=1}^{\infty} (\frac{-4}{5})^n$

(g) $\sum_{n=1}^{\infty} \frac{n^2}{\sqrt{n^3+1}}$

(h) $\sum_{n=1}^{\infty} \frac{3^n}{n}$

(i) $\sum_{n=1}^{\infty} \frac{n!}{n^n}$

(j) $\sum_{n=1}^{\infty} \frac{n!}{1 \cdot 3 \cdots (2n-1)}$

(k) $1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \cdots$

(l) $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n}$

(m) $\sum_{n=1}^{\infty} \frac{n^3}{n^5+3}$

(n) $\sum_{n=1}^{\infty} \frac{n}{2^n}$

(o) $\sum_{n=1}^{\infty} n \sin(\frac{1}{n})$

(p) $\sum_{n=1}^{\infty} \frac{1}{n^2+\sqrt{n}}$

(q) $\sum_{n=1}^{\infty} \frac{n!(n+1)!}{(3n)!}$

(r) $\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln(n)}$

11. For each of the following Taylor series, determine the precise interval of convergence.

(a) $\sum_{n=1}^{\infty} \frac{(2x-5)^n}{n^2 4^n}$

(b) $\sum_{n=1}^{\infty} \frac{(x-4)^n}{5^n}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n(x-2)^n}{n3^n}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n(x-2)^n}{n^2}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{(x-1)^n}{n^2}$

(f) $\displaystyle\sum_{n=1}^{\infty} \frac{nx^n}{2^n}$

12. Write the second Taylor polynomial for $\sqrt{x}$, centered at 25. Use this polynomial to estimate $\sqrt{26}$. Also, give an estimate of the error.

13. Use the first two non-zero terms of an appropriate series to give an approximation of

$$\int_0^1 \sin(x^2)\, dx.$$

Give an estimate of the error.

14. What is the sixth Taylor polynomial for xe^{3x}, centered at 0?

15. What is $\displaystyle\lim_{x\to 0} \frac{\cos(x)-1}{e^{x^2}-1}$?

16. Which of the following is the closest value of

$$\int_0^{0.1} \cos(x^2)\, dx?$$

Justify your answer.

(A) 0.0998 (B) 0.0999 (C) 0.1000 (D) 0.1001 (E) 0.1002 (F) 20.1003

17. Which of the following is the closest value of $\cos(\frac{\pi}{5})$?

(A) $\frac{198}{200}$ (B) $\frac{199}{200}$ (C) 2 (D) $\frac{201}{200}$ (E) $\frac{202}{200}$ (F) $\frac{203}{200}$