1. Consider an alternative to the Borda method among n candidates:

   - first preference gets $n - 1$ points
   $\vdots$
   - last preference gets 0 points

   Can the winner ever be different than the winner using the original Borda method? Explain why or why not.

   Solution: The outcome will be the same. The upshot of the changing the points is that very candidate will receive exactly one less point per voter. It therefore that the raw scores will be n less than the original. This does not affect the outcome.

2. The following table provides the preferences of voters:

Number of Voters	**7**	**5**	**6**
1st **choice**	A	D	C
2nd **choice**	B	C	D
3rd **choice**	C	B	B
4th **choice**	D	A	A

   (a) Who is the plurality winner?

   Solution: A gets 7 votes, D gets 5 votes, and C gets 6 votes. A wins.

   (b) Who is the instant runoff winner?

   Solution: At the end of round 1, B is eliminated. At the end of round 2, D is eliminated. At this point, C has a majority, and wins.

   (c) Who is the Borda method winner?

   Solution:
   A gets $7 \cdot 4 + 11 \cdot 1 = 39$ points
   B gets $7 \cdot 3 + 11 \cdot 2 = 43$ points
   C gets $6 \cdot 4 + 5 \cdot 3 + 7 \cdot 2 = 53$ points
   D gets $5 \cdot 4 + 6 \cdot 3 + 7 \cdot 1 = 45$ points
   C wins.

   (d) Who will win if one uses the method of pairwise comparisons?

   Solution: In one-on-one compettitions, C beats all other candidates. C therefore the winner.

3. Consider an election with 3 candidates, A, B, and C. Describe a scenario in which the ranked-vote, instant runoff winner is NOT the same as the Condorcet winner. (Your example will therefore show that ranked-vote, instant runoff does not satisfy the Condorcet criterion.)

   Solution:
   Such an election could look like:

Number of Voters	3	2	2
1st **choice**	A	B	C
2nd **choice**	C	C	A
3rd **choice**	B	A	B

   Note that A wins the runoff election. However, C is the Condorcet winner.