

Please make your answers and clear and complete as possible. When asked to explain answers, **please write in complete sentences**. In some cases you will be instructed to leave your numerical answers written in a form that could be entered in a calculator.



**Question 1.** (8 points) In a game of tennis, player  $A$  can choose to send the ball “down the line” (call this move “DL”) or cross-court (call this move “CC”). The opponent, player  $B$ , will try to anticipate either DL or CC. This is indicated in the following game table, where the payoffs are to player  $A$ , who is the row player (measured as the success rate, out of 100).

	DL	CC
DL	50	80
CC	90	20

- (a) Find the optimal mixed strategy for player  $A$ .
- (b) What is the expected value of the game for Player  $A$ , if the optimal mixed strategy is used?
- (c) In your own words, what is the reason for setting the expected values in (a) equal to each other?

**Question 2.** (6 points) The following is a  $4 \times 4$  game table, where the payoffs are to the row player.

3	0	-2	1
1	-4	1	0
2	-3	1	3
0	0	2	1

- (a) Eliminate any dominated strategies for the row player and the column player by crossing them out in the table above.
- (b) Determine if the game has a saddle point. If so, what is it/are they?

(c) Explain why a saddle point is always a Nash equilibrium.

**Question 3.** (8 points) Consider the following game table for a two-player game. A payoff of  $(4, 2)$ , for instance, means the row player gets 4 and the column player gets 2. In this case, both players want a larger number.

$(3,3)$	$(1,4)$
$(4,2)$	$(2,1)$

(a) Find any dominant strategies for either player.

(b) Find any Nash equilibrium/equilibria.

(c) What game or games that we discussed in class does this resemble? (Be sure to include a discussion of possibilities for cooperation and betrayal.)

**Question 4.** (8 points) Please answer the following questions on voting; short answers of 1-2 sentences are sufficient.

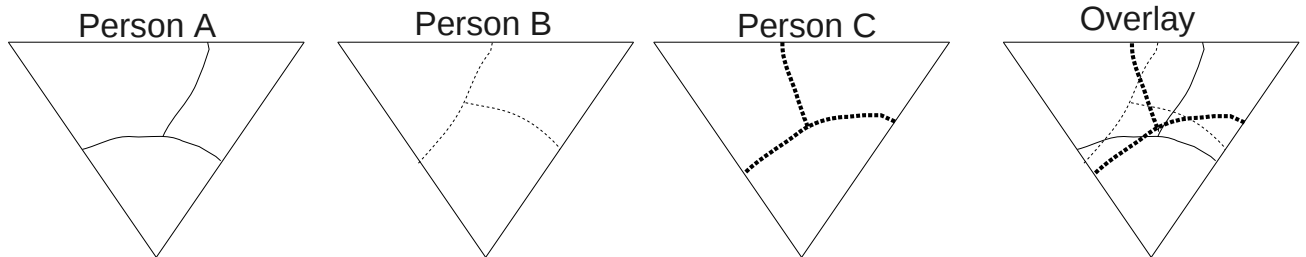
- (a) Is the Borda count system manipulable (from the perspective of a voter)?
  
  
  
  
  
  
  
  
  
  
- (b) What is the main flaw with Condorcet's voting method?
  
  
  
  
  
  
  
  
  
  
- (c) For a multi-candidate election, does the Pareto condition say that if everyone prefers candidate  $X$  over candidate  $Y$ , then  $X$  must win the election?
  
  
  
  
  
  
  
  
  
  
- (d) Is approval voting clone-independent?

**Question 5.** (8 points) You're driving from Philadelphia to Wilmington. You have the choice to either take the highway, or use backroads. Using backroads will definitely take 55 minutes. The highway will take 30 minutes IF the traffic is good, which you estimate is 50% likely. If the traffic is bad, you have the option of staying in the bad traffic (which means it will take 70 minutes in total to arrive), or to try an alternate route. If the alternate route has good traffic (60% likely), the total trip will take 60 minutes; otherwise it will take 100 minutes.

Draw and solve a decision tree to help you decide which route to take. What is your best plan of action (from start to finish)? You may wish to use the reverse side of this sheet.

**Question 6.** (6 points) The figures below are preference diagrams for three people trying to divide up a triangular cake. As usual, the goal is to find a point on the cake, make straight cuts out to the corners, and decide who gets the North piece, the West piece, and the East piece.

In the overlaid diagram *very carefully* indicate all points from which the cake can be cut from that guarantees each person has a different first choice. Also: **who gets which piece?**



**Question 7.** (6 points) The board of education has one chairperson (who has 3 votes), one vice-chair (who has 2 votes), and three regular members (who have 1 vote each). To pass a measure, they need 6 votes.

(a) Are there any dictators on the board? Why or why not?

(b) Does anyone on the board have veto power? Why or why not?

(c) Compute the Shapley–Shubik power index for the chairperson. (Note:  $5! = 120$ , and you need not simplify any fractions. It may be helpful to use the notation “C” for the chair, “V” for the vice-chair, and “M” for a regular member.)