Administration

- please fill out questionnaire

Syllabus

- office hours (Mon/Wed 11–12)

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- textbook

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- questions?

A Game Show:

▶ You choose to either roll or not roll a six-sided die. I'll pay you
$10,000 times the value of the die OR $30,000 if you do not
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- ▶ Do you ROLL or NOT?

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- Do you ROLL or NOT?
- What is the dilemma here?

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- An **event** is some collection of outcomes.

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- Note that $P(A)$ is always a number between 0 and 1.

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- We want the **expected payout** of the experiment.

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- What is the average amount that we will win per roll?

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- This is called the **expected payout** or **expected value**.
- Formally, if all outcomes are $A_1, \ldots, A_n$, the expected values is $A_1 \cdot P(A_1) + A_2 \cdot P(A_2) + \cdots + A_n \cdot P(A_n)$

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- An interpretation: You can't make $35,000$ in a single roll, but in the long run, rolling will make you more money.