

Administration

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- ▶ Not always best to choose using expected value.

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- ▶ If the stocks cost the same, which will bring in more money over time?
 - ▶ Not enough information!

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- ▶ On average, Apple pays more than Google
- ▶ What part of the above question is unnecessary?

Moral:

Expected value gives a way to evaluate decision alternatives

PROVIDED:

- ▶ The probabilities of events are known.
- ▶ The same experiment is repeated many times.

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 - ▶ Where do percentages come from?
 - ▶ Precedence, aspects of case, knowledge of judge...

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- ▶ Work from right to left (“**Decision Rollback**”)
- ▶ Compute expected values of chance nodes
- ▶ Compute expected values of decision nodes and eliminate bad choices

Product Decision

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Land Purchase

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