Playing Cards

- See the Handout #3.

From Last Time:

- We also had three axioms of probabilities:

From Last Time:

- We also had three axioms of probabilities:
  - Axiom 1: The probability of any event is a nonnegative number.
  - Axiom 2: The certain event S has probability 1.

From Last Time:

- We also had three axioms of probabilities:
  - Axiom 1: The probability of any event is a nonnegative number.
  - Axiom 2: The certain event S has probability 1.
  - Axiom 3 (Addition Rule): If A and B are mutually exclusive events, then
$$P(A \cup B) = P(A) + P(B).$$

From Last Time:

- We also had three axioms of probabilities:
  - Axiom 1: The probability of any event is a nonnegative number.
  - Axiom 2: The certain event S has probability 1.
  - Axiom 3 (Addition Rule): If A and B are mutually exclusive events, then
$$P(A \cup B) = P(A) + P(B).$$
  - Draw a Venn diagram for Axiom 3.

Example: The Addition Rule

- Let the experiment be checking the high temperature on a randomly chosen day in July.

Example: The Addition Rule

- Let the experiment be checking the high temperature on a randomly chosen day in July.
  - Let X be the event "the high is between 80 and 89 degrees," and say $P(X) = 0.5$.
  - Let Y be the event "the high is between 90 and 99 degrees," and say $P(Y) = 0.2$.
  - Let Z be the event "the high is between 85 and 94 degrees," and say $P(Z) = 0.4$.

Example: The Addition Rule

- Let the experiment be checking the high temperature on a randomly chosen day in July.
  - Let X be the event "the high is between 80 and 89 degrees," and say $P(X) = 0.5$.
  - Let Y be the event "the high is between 90 and 99 degrees," and say $P(Y) = 0.2$.
  - Let Z be the event "the high is between 85 and 94 degrees," and say $P(Z) = 0.4$.
- What is the probability that the high is between 80 and 99?

Example: The Addition Rule

- Let the experiment be checking the high temperature on a randomly chosen day in July.
  - Let X be the event "the high is between 80 and 89 degrees," and say $P(X) = 0.5$.
  - Let Y be the event "the high is between 90 and 99 degrees," and say $P(Y) = 0.2$.
  - Let Z be the event "the high is between 85 and 94 degrees," and say $P(Z) = 0.4$.

Example: The Addition Rule

- Let the experiment be checking the high temperature on a randomly chosen day in July.
  - Let X be the event "the high is between 80 and 89 degrees," and say $P(X) = 0.5$.
  - Let Y be the event "the high is between 90 and 99 degrees," and say $P(Y) = 0.2$.
  - Let Z be the event "the high is between 85 and 94 degrees," and say $P(Z) = 0.4$.
- What is the probability that the high is between 80 and 94?

Inclusion–Exclusion Rule

- Why is $P(A \cup B)$ not equal to $P(A) + P(B)$ in general?

Inclusion–Exclusion Rule

- Why is $P(A \cup B)$ not equal to $P(A) + P(B)$ in general?
- Draw diagram,

Inclusion–Exclusion Rule

- Why is $P(A \cup B)$ not equal to $P(A) + P(B)$ in general?
- Draw diagram, $A \cap B$ is counted twice in $P(A) + P(B)$, so you need to subtract it!

Inclusion–Exclusion Rule

- Why is $P(A \cup B)$ not equal to $P(A) + P(B)$ in general?
- Draw diagram, $A \cap B$ is counted twice in $P(A) + P(B)$, so you need to subtract it!

Inclusion–Exclusion Rule

If A and B are events (not necessarily mutually exclusive), then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

Inclusion–Exclusion Rule

- Let the experiment be checking the high temperature on a randomly chosen day in July.
  - Let X be the event "the high is between 80 and 89 degrees," and say $P(X) = 0.5$.
  - Let Y be the event "the high is between 90 and 99 degrees," and say $P(Y) = 0.2$.
  - Let Z be the event "the high is between 85 and 94 degrees," and say $P(Z) = 0.4$.
  - Let W be the event "the high is between 85 and 89 degrees," and say $P(W) = 0.2$.

Inclusion–Exclusion Rule

- Let the experiment be checking the high temperature on a randomly chosen day in July.
  - Let X be the event "the high is between 80 and 89 degrees," and say $P(X) = 0.5$.
  - Let Y be the event "the high is between 90 and 99 degrees," and say $P(Y) = 0.2$.
  - Let Z be the event "the high is between 85 and 94 degrees," and say $P(Z) = 0.4$.
  - Let W be the event "the high is between 85 and 89 degrees," and say $P(W) = 0.2$.
- What is the probability that the high is between 80 and 94?

Example: Sickness

- Suppose the Center for Disease Control (CDC) predicts a 50% chance that an individual will contract a cold in the next year, and a 20% chance that an invidual will contract the flu.

Example: Sickness

- Suppose the Center for Disease Control (CDC) predicts a 50% chance that an individual will contract a cold in the next year, and a 20% chance that an invidual will contract the flu.
- Suppose the CDC also predicts a 15% chance of a person contracting both a cold AND the flu.

Example: Sickness

- Suppose the Center for Disease Control (CDC) predicts a 50% chance that an individual will contract a cold in the next year, and a 20% chance that an invidual will contract the flu.
- Suppose the CDC also predicts a 15% chance of a person contracting both a cold AND the flu.
- Find the probability of an individual contracting a cold OR the flu.

Example: Sickness

- Suppose the Center for Disease Control (CDC) predicts a 50% chance that an individual will contract a cold in the next year, and a 20% chance that an invidual will contract the flu.
- Suppose the CDC also predicts a 15% chance of a person contracting both a cold AND the flu.
- Find the probability of an individual contracting a cold OR the flu.
- Note: in this example, to find $P(C \cup F)$, we needed to know not just $P(C)$ and $P(F)$, but also $P(C \cap F)$.

Rules of Probability

- Using the axioms, one can *deduce* some more rules of probabilities.

Impossible Event Rule

$P(\emptyset) =$

Rules of Probability

- Using the axioms, one can *deduce* some more rules of probabilities.

Impossible Event Rule

$P(\emptyset) = 0$. That is, the probability of the impossible event is zero.

Rules of Probability

- Using the axioms, one can *deduce* some more rules of probabilities.

Impossible Event Rule

$P(\emptyset) = 0$. That is, the probability of the impossible event is zero.

General Addition Rule:

If $E_1, E_2, \ldots E_n$ are mutually exclusive events (meaning $E_i \cap E_j = \emptyset$ for all $i \neq j$), then

$$P(E) =$$

Rules of Probability

- Using the axioms, one can *deduce* some more rules of probabilities.

Impossible Event Rule

$P(\emptyset) = 0$. That is, the probability of the impossible event is zero.

General Addition Rule:

If $E_1, E_2, \ldots E_n$ are mutually exclusive events (meaning $E_i \cap E_j = \emptyset$ for all $i \neq j$), then

$$P(E) = P(E_1) + P(E_2) + \ldots + P(E_n),$$

where $E = E_1 \cup E_2 \cup \ldots \cup E_n$ is the event that at least one of the E_i occurs.

Rules of Probability

Subset Rule

If A and B are events such that every outcome in A is also in B, then:

Rules of Probability

Subset Rule

If A and B are events such that every outcome in A is also in B, then: $P(A) \leq P(B)$

- The idea: more ways B can occur, so B is more likely. Draw diagram.