Friendship Paradox

- Take a group of people. For each person, consider the numbers:
  - number of friends the person has,
  - average number of friends that friends of the person has.

Friendship Paradox

- Take a group of people. For each person, consider the numbers:
  - number of friends the person has,
  - average number of friends that friends of the person has.
- For most people, the second number is at least as large as the first.

Friendship Paradox

- Take a group of people. For each person, consider the numbers:
  - number of friends the person has,
  - average number of friends that friends of the person has.
- For most people, the second number is at least as large as the first.
- You're more likely to be friends with someone with a lot of friends...

Friendship Paradox

- Take a group of people. For each person, consider the numbers:
  - number of friends the person has,
  - average number of friends that friends of the person has.
- For most people, the second number is at least as large as the first.
- You're more likely to be friends with someone with a lot of friends...
- and less likely to be friends with someone with few friends.

Friendship Paradox

- Take a group of people. For each person, consider the numbers:
  - number of friends the person has,
  - average number of friends that friends of the person has.
- For most people, the second number is at least as large as the first.
- You're more likely to be friends with someone with a lot of friends...
- and less likely to be friends with someone with few friends.
- Average Facebook user has 245 friends. Average friend on Facebook has 359 friends.

Friendship Paradox

- Take a group of people. For each person, consider the numbers:
  - number of friends the person has,
  - average number of friends that friends of the person has.
- For most people, the second number is at least as large as the first.
- You're more likely to be friends with someone with a lot of friends...
- and less likely to be friends with someone with few friends.
- Average Facebook user has 245 friends. Average friend on Facebook has 359 friends.
- "Everyone you follow or who follows you has more friends and followers than you" holds for $>$ 98% of Twitter users.

Coins and Dice

- Suppose you do an experiment where you flip a coin and roll a die.

Coins and Dice

- Suppose you do an experiment where you flip a coin and roll a die.
  - $T3$ is a sample outcome.

Coins and Dice

- Suppose you do an experiment where you flip a coin and roll a die.
  - $T3$ is a sample outcome.
- Sample space is:

$$\{H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6\}$$

Coins and Dice

- Suppose you do an experiment where you flip a coin and roll a die.
  - $T3$ is a sample outcome.
- Sample space is:

$$\{H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6\}$$

- Let $A = \{\text{coin shows heads}\}$. Let $B = \{\text{die shows 3}\}$.

Coins and Dice

- Suppose you do an experiment where you flip a coin and roll a die.
  - $T3$ is a sample outcome.
- Sample space is:

$$\{H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6\}$$

- Let $A = \{\text{coin shows heads}\}$. Let $B = \{\text{die shows 3}\}$.
- If you know A occurs, does it affect the probability of B?

Independent Events

Definition

In an experiment, events A and B are **independent** if knowledge that A occurs does not affect the probability that B occurs.

Examples

- Let C be the event that it will be cloudy tomorrow, and let R be the event that it will rain tomorrow.

Examples

- Let C be the event that it will be cloudy tomorrow, and let R be the event that it will rain tomorrow.
  - Knowing C makes R more likely. Knowing R makes C more likely as well.

Examples

- Let C be the event that it will be cloudy tomorrow, and let R be the event that it will rain tomorrow.
  - Knowing C makes R more likely. Knowing R makes C more likely as well.
- Flip a coin twice. Let $A=\{$heads on first flip$\}$, and $B=\{$tails on second flip$\}$.

Examples

- Let C be the event that it will be cloudy tomorrow, and let R be the event that it will rain tomorrow.
  - Knowing C makes R more likely. Knowing R makes C more likely as well.
- Flip a coin twice. Let $A =${heads on first flip}, and $B =${tails on second flip}.
  - A and B are independent.

Examples

- Let C be the event that it will be cloudy tomorrow, and let R be the event that it will rain tomorrow.
  - Knowing C makes R more likely. Knowing R makes C more likely as well.
- Flip a coin twice. Let A ={heads on first flip}, and B ={tails on second flip}.
  - A and B are independent.
- Randomly select a person. Let H = {person has heart disease}, and let T = { person is under age 30}.

Examples

- Let C be the event that it will be cloudy tomorrow, and let R be the event that it will rain tomorrow.
  - Knowing C makes R more likely. Knowing R makes C more likely as well.
- Flip a coin twice. Let $A =$\{heads on first flip\}, and $B =$\{tails on second flip\}.
  - A and B are independent.
- Randomly select a person. Let $H = $ \{person has heart disease\}, and let $T = $ \{ person is under age 30\}.
  - Knowing H makes T less likely; knowing T makes H less likely.

Independent Events

- Take a randomly selected person from the U.S.

Independent Events

- Take a randomly selected person from the U.S.
- Say $P(\text{is female}) = 0.515$, and
- $P(\text{green eyes}) = \frac{1}{8}$.

Independent Events

- Take a randomly selected person from the U.S.
- Say $P(\text{is female}) = 0.515$, and
- $P(\text{green eyes}) = \frac{1}{8}$.
- Assume these events are independent.

Independent Events

- Take a randomly selected person from the U.S.
- Say $P(\text{is female}) = 0.515$, and
- $P(\text{green eyes}) = \frac{1}{8}$.
- Assume these events are independent.
- How likely is it that a person is female AND has green eyes?

Independent Events

- Take a randomly selected person from the U.S.
- Say $P(\text{is female}) = 0.515$, and
- $P(\text{green eyes}) = \frac{1}{8}$.
- Assume these events are independent.
- How likely is it that a person is female AND has green eyes?
- Presumably the answer is $\frac{1}{8} \times 0.515$, which is ≈ 0.065.

Multiplication Rule for Independent Events

Multiplication Rule for Independent Events

If A and B are independent events, then

$$P(A \cap B) = P(A) \cdot P(B)$$

Multiplication Rule for Independent Events

Multiplication Rule for Independent Events

If A and B are independent events, then

$$P(A \cap B) = P(A) \cdot P(B)$$

WARNING: independent events are different than mutually exclusive events!

Cards

- Draw a card at random from a standard deck (then replace it and shuffle).

Cards

- Draw a card at random from a standard deck (then replace it and shuffle).
- Repeat once.

Cards

- Draw a card at random from a standard deck (then replace it and shuffle).
- Repeat once.
- What is probability that both cards are spades?

Cards

- Draw a card at random from a standard deck (then replace it and shuffle).
- Repeat once.
- What is probability that both cards are spades?
- What is the probability that both cards are the same suit?

Winning the Lottery Twice!

- Say the probability that a lottery ticket wins a \$1 million jackpot is $\frac{1}{1{,}000{,}000}$.

Winning the Lottery Twice!

- Say the probability that a lottery ticket wins a $1 million jackpot is $\frac{1}{1{,}000{,}000}$.
- Buy two tickets (spaced months apart, say). What is probability of both being winners?

Winning the Lottery Twice!

- Say the probability that a lottery ticket wins a $1 million jackpot is $\frac{1}{1{,}000{,}000}$.
- Buy two tickets (spaced months apart, say). What is probability of both being winners?
- Incredibly unlikely- but it happened to Ernest Pullen.

Reading: Ernest Pullen

- Why might it be reasonable to expect multiple lottery winners:

Reading: Ernest Pullen

- Why might it be reasonable to expect multiple lottery winners:
  - Question is: what are chances that SOMEONE wins twice, not that YOU win twice.

Reading: Ernest Pullen

- Why might it be reasonable to expect multiple lottery winners:
  - Question is: what are chances that SOMEONE wins twice, not that YOU win twice.
  - People might buy more than one ticket.

Reading: Ernest Pullen

- Why might it be reasonable to expect multiple lottery winners:
  - Question is: what are chances that SOMEONE wins twice, not that YOU win twice.
  - People might buy more than one ticket.
  - Pullen may have used $100,000s of his first winnings to buy more tickets

Reading: Ernest Pullen

- Why might it be reasonable to expect multiple lottery winners:
  - Question is: what are chances that SOMEONE wins twice, not that YOU win twice.
  - People might buy more than one ticket.
  - Pullen may have used $100,000s of his first winnings to buy more tickets
- Lottery spokesman, on likelihood of winning twice: *"Because they're independent games, it is impossible to calculate the odds."* Anything wrong with this?

DNA Evidence

- Criminal trial. Defendant's DNA matches blood at crime scene.

DNA Evidence

- Criminal trial. Defendant's DNA matches blood at crime scene.
- Say $P(\{\text{accidental match}\}) = \frac{1}{100{,}000{,}000}$.

DNA Evidence

- Criminal trial. Defendant's DNA matches blood at crime scene.
- Say $P(\{\text{accidental match}\}) = \frac{1}{100{,}000{,}000}$.
- Many experts: $P(\{\text{laboratory error}\}) = \frac{1}{100}$.

DNA Evidence

- Criminal trial. Defendant's DNA matches blood at crime scene.
- Say $P(\{\text{accidental match}\}) = \frac{1}{100{,}000{,}000}$.
- Many experts: $P(\{\text{laboratory error}\}) = \frac{1}{100}$.
- Jury is interested in...

DNA Evidence

- Criminal trial. Defendant's DNA matches blood at crime scene.
- Say $P(\{\text{accidental match}\}) = \frac{1}{100{,}000{,}000}$.
- Many experts: $P(\{\text{laboratory error}\}) = \frac{1}{100}$.
- Jury is interested in...
  $P(\{\text{accidental match}\} OR \{\text{laboratory error}\})$

DNA Evidence

- Criminal trial. Defendant's DNA matches blood at crime scene.
- Say $P(\{\text{accidental match}\}) = \frac{1}{100{,}000{,}000}$.
- Many experts: $P(\{\text{laboratory error}\}) = \frac{1}{100}$.
- Jury is interested in...
  $P(\{\text{accidental match}\} OR \{\text{laboratory error}\}) = \frac{100{,}000{,}099}{100{,}000{,}000{,}000}$.

Be Careful Multiplying Probabilities

- Look at the eye color of a randomly selected person in the U.S.

Be Careful Multiplying Probabilities

- Look at the eye color of a randomly selected person in the U.S.
- Say $P(\text{left eye is green}) = 0.12$, and

Be Careful Multiplying Probabilities

- Look at the eye color of a randomly selected person in the U.S.
- Say $P(\text{left eye is green}) = 0.12$, and
- $P(\text{right eye is green}) = 0.12$

Be Careful Multiplying Probabilities

- Look at the eye color of a randomly selected person in the U.S.
- Say $P(\text{left eye is green}) = 0.12$, and
- $P(\text{right eye is green}) = 0.12$
- What is $P(\text{left and right eye are green})$?

Be Careful Multiplying Probabilities

- Look at the eye color of a randomly selected person in the U.S.
- Say $P(\text{left eye is green}) = 0.12$, and
- $P(\text{right eye is green}) = 0.12$
- What is $P(\text{left and right eye are green})$?
  - NOT 0.0144.

Be Careful Multiplying Probabilities

- Look at the eye color of a randomly selected person in the U.S.
- Say $P(\text{left eye is green}) = 0.12$, and
- $P(\text{right eye is green}) = 0.12$
- What is $P(\text{left and right eye are green})$?
  - NOT 0.0144.
  - Close to 0.12.