Announcements

- Office hours are now 2-4 on Tuesdays.
- Question 4: What is the probability that AT LEAST one die lands on 1?

Baseball

- Suppose that a baseball player has a batting average of 0.333.

Baseball

- Suppose that a baseball player has a batting average of 0.333.
- This means that the probability that he/she hits the ball is 0.333 in each at-bat.

Baseball

- Suppose that a baseball player has a batting average of 0.333.
- This means that the probability that he/she hits the ball is 0.333 in each at-bat.
- Assume the attempts are independent. (Reasonable?)

Baseball

- Suppose that a baseball player has a batting average of 0.333.
- This means that the probability that he/she hits the ball is 0.333 in each at-bat.
- Assume the attempts are independent. (Reasonable?)
- If the player strikes out 10 times in a row, are they "due for a hit"?

Law of Averages Fallacy

False:

Law of Averages Fallacy

(For instance) If you flip a coin and get 100 tails is a row, the probability that the next flip is a heads is greater than 0.5.

Law of Averages Fallacy

False:

Law of Averages Fallacy

(For instance) If you flip a coin and get 100 tails is a row, the probability that the next flip is a heads is greater than 0.5.

(False because the trials are independent; the coin knows no history.)

The People of the State of California v. Collins

- Famous criminal trial from 1968, in L.A. (robbery of eldery woman)

The People of the State of California v. Collins

- Famous criminal trial from 1968, in L.A. (robbery of eldery woman)
- The two robbers had the following characteristics:

The People of the State of California v. Collins

- Famous criminal trial from 1968, in L.A. (robbery of eldery woman)
- The two robbers had the following characteristics:
- According to victim:
  - perpetrator: white female with blonde hair in ponytail.

The People of the State of California v. Collins

- Famous criminal trial from 1968, in L.A. (robbery of eldery woman)
- The two robbers had the following characteristics:
- According to victim:
  - perpetrator: white female with blonde hair in ponytail.
- According to other witness:
  - getaway driver: black male with beard and mustache,

The People of the State of California v. Collins

- Famous criminal trial from 1968, in L.A. (robbery of eldery woman)
- The two robbers had the following characteristics:
- According to victim:
  - perpetrator: white female with blonde hair in ponytail.
- According to other witness:
  - getaway driver: black male with beard and mustache,
  - in a yellow get-away vehicle

The People of the State of California v. Collins

- Prosecution called as expert witness "an instructor of mathematics," who presented the following table:

The People of the State of California v. Collins

- Prosecution called as expert witness "an instructor of mathematics," who presented the following table:

Characteristic	probability
Black man with beard	1/10
Man with moustache	1/4
White woman with pony tail	1/10
White woman with blonde hair	1/3
Yellow motor car	1/10
Interracial couple in car	1/1000

The People of the State of California v. Collins

- Prosecution called as expert witness "an instructor of mathematics," who presented the following table:

Characteristic	probability
Black man with beard	1/10
Man with moustache	1/4
White woman with pony tail	1/10
White woman with blonde hair	1/3
Yellow motor car	1/10
Interracial couple in car	1/1000

- "Expert" claimed: to find the probability that a random couple matches all these characteristics, you multiply the invidual probabilities.

The People of the State of California v. Collins

- Couple was found guilty!

The People of the State of California v. Collins

- Couple was found guilty!
- If you were the defense, how might you respond to this?

The People of the State of California v. Collins

- Couple was found guilty!
- If you were the defense, how might you respond to this?
  - Multiplying probabilities works only for *independent* events!

The People of the State of California v. Collins

- Couple was found guilty!
- If you were the defense, how might you respond to this?
  - Multiplying probabilities works only for *independent* events!
- Maybe 1 in a million couples match these characteristics, so easily three such couples in L.A. area.

The People of the State of California v. Collins

- Couple was found guilty!
- If you were the defense, how might you respond to this?
  - Multiplying probabilities works only for *independent* events!
- Maybe 1 in a million couples match these characteristics, so easily three such couples in L.A. area.
- Without more information, the probability is only about 1 in 3 that they are guilty.

The People of the State of California v. Collins

- Conviction later set aside by California Supreme Court: “trial by mathematics”.

The People of the State of California v. Collins

- Conviction later set aside by California Supreme Court: "trial by mathematics".
- Even without using specific numbers, this type of reasoning is used by lawyers, judges, and juries!

The People of the State of California v. Collins

- Conviction later set aside by California Supreme Court: "trial by mathematics".
- Even without using specific numbers, this type of reasoning is used by lawyers, judges, and juries!
- Example of "prosecutor's fallacy": more later.

Conditional Probability

- Let A and B be events. $P(A|B)$ means "the probability of A, given B"

Conditional Probability

- Let A and B be events. $P(A|B)$ means "the probability of A, given B"
- $P(\text{rain}|\text{cloudy}) = ?$

Conditional Probability

- Let A and B be events. $P(A|B)$ means "the probability of A, given B"
- $P(\text{rain}|\text{cloudy}) = ?$
- $P(\text{cloudy}|\text{rain}) = ?$

Dice

- Roll two dice. What is the probability that the dice add up to 10 or greater?

Dice

- Roll two dice. What is the probability that the dice add up to 10 or greater?
- What is this probability if you're given that the first die shows a six?

Boys and Girls

- Suppose a family has two children. Four outcomes (for gender), equally likely:

$$\{BB, BG, GB, GG\}$$

Boys and Girls

- Suppose a family has two children. Four outcomes (for gender), equally likely:

$$\{BB, BG, GB, GG\}$$

- What is the probability of having two boys (event X)?

Boys and Girls

- Suppose a family has two children. Four outcomes (for gender), equally likely:

$$\{BB, BG, GB, GG\}$$

- What is the probability of having two boys (event X)?
- What is the probability of having at least one boy (event Y)?

Boys and Girls

- Suppose a family has two children. Four outcomes (for gender), equally likely:

$$\{BB, BG, GB, GG\}$$

- What is the probability of having two boys (event X)?
- What is the probability of having at least one boy (event Y)?
- Find $P(X|Y)$. This means...

Boys and Girls

- Suppose a family has two children. Four outcomes (for gender), equally likely:

$$\{BB, BG, GB, GG\}$$

- What is the probability of having two boys (event X)?
- What is the probability of having at least one boy (event Y)?
- Find $P(X|Y)$. This means...
- Find $P(Y|X)$. This means...

Two tests

- A teacher gives two tests. 60% of the students passed them both.

Two tests

- A teacher gives two tests. 60% of the students passed them both.
- 80% of the students passed the first test.

Two tests

- A teacher gives two tests. 60% of the students passed them both.
- 80% of the students passed the first test.
- Question: among those who passed the first test, what fraction also passed the second test?

Multiplication rule

Multiplication rule for conditional probabilities

For any events A and B, we have

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

so

$$P(A \cap B) = P(B|A) \cdot P(A)$$

Facebook and iPhones

- Pick a U.S. citizen at random.

Facebook and iPhones

- Pick a U.S. citizen at random.
- $P(\{\text{Facebook user}\}) \approx 0.5$

Facebook and iPhones

- Pick a U.S. citizen at random.
- $P(\{\text{Facebook user}\}) \approx 0.5$
- Among U.S. Facebook users, say 8% have an iPhone.

Facebook and iPhones

- Pick a U.S. citizen at random.
- $P(\{\text{Facebook user}\}) \approx 0.5$
- Among U.S. Facebook users, say 8% have an iPhone.
- Question: What fraction of U.S. citizens are on facebook AND have an iPhone?

Back to Independent Events

- Another way of understanding independent events:

Back to Independent Events

- Another way of understanding independent events:
- A and B are independent if $P(A|B) = P(A)$.

Back to Independent Events

- Another way of understanding independent events:
- A and B are independent if $P(A|B) = P(A)$.
- That is, A and B are independent if knowledge of B does not affect A.