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  - Do we know what percent of the population carries the genetic marker for Cancer B?

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  - 2% of uninfected people also test positive.
- If you test positive, how likely is it that you have the disease?

Testing for a Disease

- The population consists of four groups:
  - True Positive: diseased people that test positive.
  - True Negative: healthy people that test negative.
  - False Positive: healthy people that test positive.
  - False Negative: diseased people that test negative.

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So if you test positive for the disease, you have a 4.7% chance of having the disease.