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Life Expectancy

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Life Expectancy

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- And a 1% chance that they will live to be 100.
  - What is the probability that someone who lives to be 90 will live to be 100?

Bayes' Formula

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- NOTE: There was an error in the formula last time!!

Testing for a Disease - Revisited

- Consider a disease that affects $\frac{1}{1000}$ people.

Testing for a Disease - Revisited

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- A test produces the results:
  - 99% of infected people test positive.
    "The test is 99% *positive"*
  - 2% of uninfected people also test positive.

Testing for a Disease - Revisited

- Consider a disease that affects $\frac{1}{1000}$ people.
- A test produces the results:
  - 99% of infected people test positive.
    "The test is 99% *positive"*
  - 2% of uninfected people also test positive.
- If you test positive, how likely is it that you have the disease?

Testing for a Disease - Revisited

- Let D be the event that the person has the disease.
- Let T be the even that the person tests positive.

Testing for a Disease - Revisited

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- We want to know:
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- Using Bayes' Formula, we get $P(D|T) = .047$.
- So if you test positive for the disease, you have a 4.7% chance of having the disease.

Testing for a Disease

Another way to see this. Consider a sample population of $100,000$.

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	Tests Positive	Tests Negative
Has Disease		

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Healthy		

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Another way to see this. Consider a sample population of $100,000$.

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Healthy	1998	

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	Tests Positive	Tests Negative
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Another way to see this. Consider a sample population of $100,000$.

	Tests Positive	Tests Negative
Has Disease	99	1
Healthy	1998	97902

So there are so many more people are a false positive than people who are a true positive.

Testing for a Disease

- How do we rule out false positives?

Testing for a Disease

- How do we rule out false positives?
  - Test Again.

Testing for a Disease

- How do we rule out false positives?
  - Test Again.
  - See Handout #4.