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- After making your choice, the host opens one of the other doors, and reveals a goat.
- You are then given the choice of changing your choice to the other remaining closed door.
- Should you switch doors or stick with your first choice? (Does it matter?)

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- The revealed goat does not change this probability
- The other door must have probability $\frac{2}{3}$ of being the correct door.
- Alternatively, switching essentially chooses two doors.

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- Label the doors A, B, and C.
  - Suppose you choose door A.
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- Label the doors A, B, and C.
  - Suppose you choose door A.
  - Suppose the host opens door C.
- Let A, B and C be the events that the cash is behind the corresponding door.
- Let I be the event that the host opens door C.

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  - $P(A) = P(B) = P(C) =$

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- We know:
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  - $P(I|A) = \frac{1}{2}$.
  - $P(I|B) = 1$.

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  - $P(I) =$

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  - $P(I|A) = \frac{1}{2}$.
  - $P(I|B) = 1$.
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  - $P(I) = P(I \cap A) + P(I \cap B) + P(I \cap C) = \frac{1}{2}$.

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- We know:
  - $P(A) = P(B) = P(C) = \frac{1}{3}$.
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  - $P(I|B) = 1$.
  - $P(I|C) = 0$.
  - $P(I) = P(I \cap A) + P(I \cap B) + P(I \cap C) = \frac{1}{2}$.
- Then

$$P(A|I) = \frac{P(I|A) \cdot P(A)}{P(I)} = \frac{1}{3}$$

and

$$P(B|I) = \frac{P(I|B) \cdot P(B)}{P(I)} = \frac{2}{3}$$

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- Defence: "an infinitesimal percentage – certainly fewer than 1 of 2,500 – of men who slap or beat their domestic partners go on to murder them."
- Is the history of domestic violence relevant?

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- Draw the Venn Diagram

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- Let A be the event that a woman is abused by her spouse.
- Let M be the event that a woman is murdered.
- Let G be the event that the woman is murdered by her spouse.

O.J. Simpson Trial

- Let A be the event that a woman is abused by her spouse.
- Let M be the event that a woman is murdered.
- Let G be the event that the woman is murdered by her spouse.
  - Defence presented $P(G|A)$.
  - Interested in $P(G|A \cap M)$.

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- In a group of $100,000$ abused women, approximately 40 will be murdered by their spouse, and 5 will be murdered by someone else.
- SO $P(M|A \cap G^c) \approx \frac{40}{45} \approx .9$