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- The variance is:

$$\sigma^2 = (0-3)^2 \cdot \frac{1}{64} + (1-3)^2 \cdot \frac{1}{64} + \cdots + (6-1)^2 \cdot \frac{1}{64} = \frac{3}{2}.$$

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- So $\mu$ is approximately 7.8 billion.

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  - We don't really know $p_{life}$, $p_{intelligence}$, and $p_{civilization}$.

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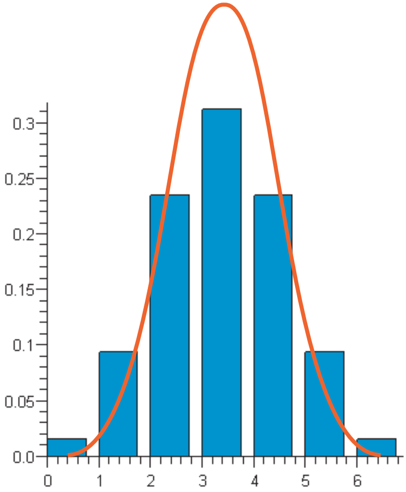

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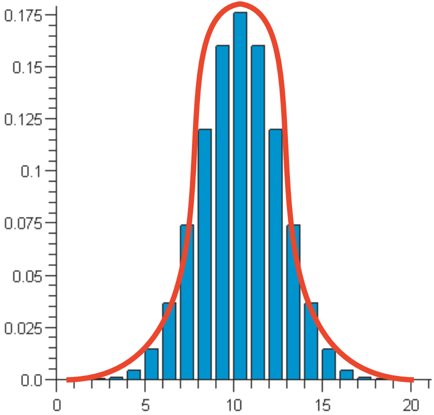

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- Moral: when $n$ gets large, the *distribution* of the number of successes looks like a bell-shaped curve.
- Where is the curve centered at?
- The standard deviation/variance measures how wide the curve is.
- The area under the curve is always 1.

# Normal Distributions

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- We need to know 2 numbers to describe the normal distribution:
  - $\mu$: the mean, where the curve is centered.
  - $\sigma$: the standard deviation, which specifies how spread out the bell is.