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  - two possible outcomes (success or failure)
  - probability of success is always the same (p)
  - the trials are independent
- other experiments:
  - "what is the probability of k successes if we do a Bernoulli trial n times?"
  - "how many successes do we expect if we do a Bernoulli trial n times?"

Multiple Choice Test

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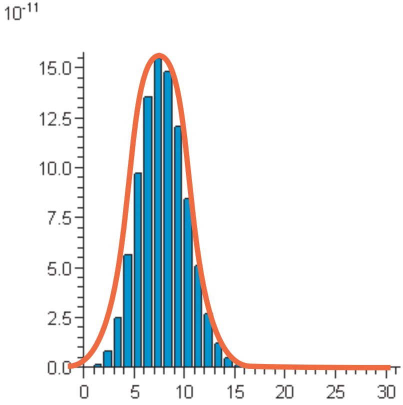
$$\left(N(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \right)$$

A Longer Test



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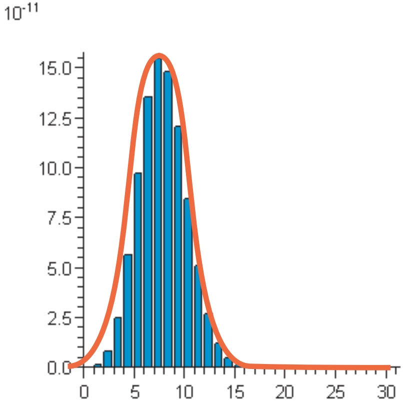

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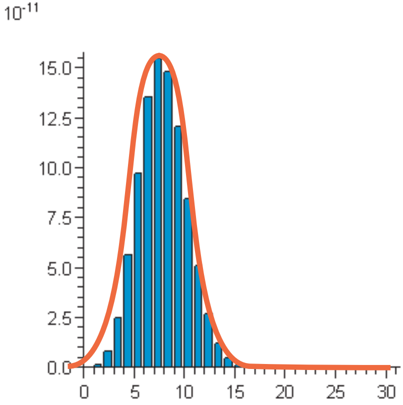

- Observe that this student doesn't stand a good chance at passing.
- Want to quantify this.

Normal Distributions

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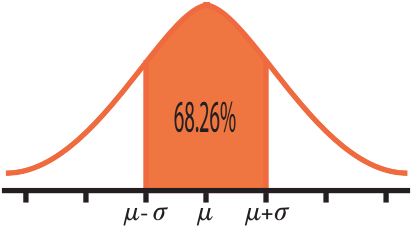

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- Under 100?
- Between 70 and 130?

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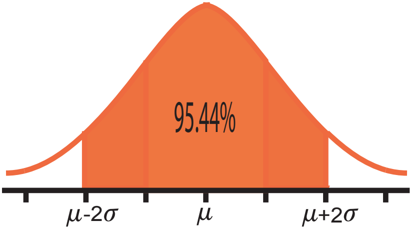

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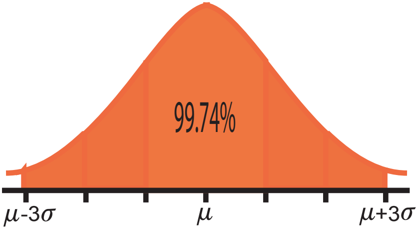
- 99.74% of the area is between $\mu - 3\sigma$ and $\mu + 3\sigma$.



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- Between 55 and 145?
- Over 145?

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  - $\sigma \approx 2.5$
- Approximately, what is the probability that the student will score under 15?

Airline Booking

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- If you sell 200 tickets, the airline will almost be under-full.
- Should incorporate these probabilities into the earlier decision theory model.

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  - Of 120?

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- To answer, define the z-score of some value x:
  - $z = \frac{x - \mu}{\sigma}$
- Back to IQs ($\mu = 100, \sigma = 15$)
  - What is the z-score of 85?
  - Of 120?
- For any normal distribution, what is the z-score of μ?

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- So the z-score just says how many standard deviations a number is above or below μ
- A z-score table tells you what percent is below x.
- What percent of the population has IQ less than or equal to 120?

Part of a z-score Table

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340
1.0	.8412	.8438	.8461	.8485	.8508	.8531	.8554	.8577
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147

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1.0	.8412	.8438	.8461	.8485	.8508	.8531	.8554	.8577
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147

Part of a z-score Table

- So $\approx$ 90.82% of the population has IQ less than 120.