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- ▶  $\hat{p}$  is normally distributed with:
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  - ▶ standard deviation  $\sigma = \sqrt{\frac{p(1-p)}{n}}$
- ▶ standard deviation is also called **sampling error**

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Get  $\sigma$  in terms of  $n$ :

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- ▶ If you quadruple  $n$ ,  $\sigma$  shrinks by 2



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How often will  $\hat{p}$  lie between .71 and .73?



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    - ▶ 99.7% of the time
  - ▶ Suppose we poll 16 times as many people as before  
How often will  $\hat{p}$  lie between .71 and .73?
    - ▶ 95% of the time

# Confidence Intervals

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  - ▶ [.625, .655] (shorthand)

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<b>2.0</b>	.9772	.9778	.9783	.9788	.9793	.9798	.9803
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<b>2.2</b>	.9861	.9864	.9868	.9871	.9875	.9878	.9881
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- ▶ To cut the margin of error in half, how many more people do we need to survey?
  - ▶ Four times as many

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- ▶ What is the m.e. for a 99.7% level of confidence?
  - ▶ First need to know  $\sigma$

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- ▶ We are 99.7% sure that the percentage of GOP voters that favor the shutdown is within 4.59% of 71%
- ▶ What is the 99.7% confidence interval?
  - ▶ Approximately [.664, .756]