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- each player has a list of moves they can make (strategies)
- there are payoffs for each eventual outcome

New Grading Scheme

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  - If you put β and the other puts α, you get F and the other gets A

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  - If you put β and the other puts α, you get F and the other gets A
  - If you both put β, you both get $B+$

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  - If you both put α, you both get *B*$-$
  - If you put β and the other puts α, you get *F* and the other gets *A*
  - If you both put β, you both get *B*$+$

The **outcome matrix**:

		Other: α	Other: β
You	α	B-,B-	A, F
	β	F, A	B+,B+

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  - A company and a union negotiating a contract
  - prosecution and defense deciding which arguments to put before the jury

Possible Payoffs

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- The outcome matrix is then:

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You	α	1, 1	3, 0
	β	0, 3	2, 2

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  - If your opponent chooses β, you're better off choosing α ($3 > 2$)

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- One strategy **strongly dominates** the other if your payoff from one is higher than the payoff from the other, regardless of others' strategies.

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- One strategy **strongly dominates** the other if your payoff from one is higher than the payoff from the other, regardless of others' strategies.
- **Moral**: you should never pick a dominated strategy

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- This time of game is called **Prisoners' Dilema**

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You	α	2, 2	1, 0
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- Is there a dominated strategy?
  - No:
    - If the other chooses α, you should also choose α
    - If the other chooses β, you should also choose β
- This is called a **coordination game**

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  - If they are rational, and they think you're rational, they'll know that you'll play α
  - So they should play α
- **Moral**: If you don't have a dominated strategy, try to predict your opponents' choice

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Fact: in Prisoners' Dilema situation, roughly 30% choose β

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  - 30 wins \$4.90