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- ▶ **Moral:** you should never pick a dominated strategy
- ▶ **Moral:** If you don't have a dominated strategy, try to predict your opponents' choice

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- ▶ Let $u_i(s_i, s_{-i})$ denote the utility/payoff for player i if players choose strategies s_i/s_{-i}
- ▶ So s_i strictly dominates s_i^* if $u_i(s_i, s_{-i}) > u_i(s_i^*, s_{-i})$ for all choices of s_{-i}

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Fabius	E	1, 1	1, 1
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 - ▶ **Moral:** you should probably never pick a weakly dominated

The Numbers Game

Review:

- ▶ Everyone in class wrote down their name and a number between 1 and 100

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 - ▶ Two thirds of the average will be ≈ 33
- ▶ Any problems?

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 - ▶ If everyone else chooses randomly, the average will be ≈ 50
 - ▶ Two thirds of the average will be ≈ 33
- ▶ Any problems?
 - ▶ If most people think this way, the average will be ≈ 33 , and so two thirds of the average will be ≈ 22

Iterative Deletion of Dominated Strategies

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- ▶ If we continue this process, everyone is left with choosing 1

The Numbers Game

- ▶ The class' numbers were:

1, 1, 1, 2.14, 15, 15, 17.9, 18, 30, 32, 34, 34, 37, 45, 45, 48, 48, 53, 80, 89

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- ▶ Two thirds of the average is 21.53

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- ▶ Two thirds of the average is 21.53
- ▶ Congratulations Andre Serrano (\$4.80)

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- ▶ Have strategies (1 – 10)
- ▶ Need payoffs: choose the % of the vote that they earn

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- ▶ If we iterate this, the candidates end up in the central positions
- ▶ This is **The Median Voter Theorem**
“Majority rule voting will select the median preference”