Review

From last time:

Strategy A strictly dominates strategy B if the payoff from A is higher than the payoff of B, regardless of others' strategies

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Moral: you should never pick a dominated strategy

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From last time:

- Strategy A strictly dominates strategy B if the payoff from A is higher than the payoff of B, regardless of others' strategies
- Moral: you should never pick a dominated strategy
- Moral: If you don't have a dominated strategy, try to predict your opponents' choice

• Let $1, 2, \ldots, n$ denote players



- ▶ Let 1, 2, ..., *n* denote players
- Let s_i denote a particular strategy of player i

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- So s_i strictly dominates s^{*}_i if u_i(s_i, s_{−i}) > u_i(s^{*}_i, s_{−i}) for all choices of s_{−i}

Hannibal wants to cross into Italy with two batallions

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There are two options:

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- There are two options:
 - Easy path along the coast

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- There are two options:
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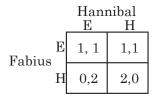
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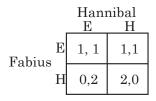
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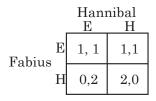


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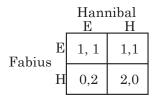
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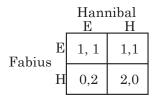
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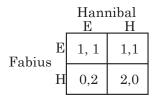


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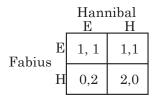


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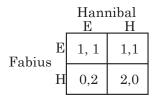
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 - Strategy s_i weakly dominates strategy s^{*}_i if the payoff from s_i is never worse than the payoff of s^{*}_i, regardless of others' strategies

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The Numbers Game

Review:

 Everyone in class wrote down their name and a number between 1 and 100

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Example:

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- Everyone in class wrote down their name and a number between 1 and 100
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- They choose 5, 30, 55

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Example:

Suppose that there are three people in class

• They choose 5, 30, 55
•
$$\frac{2}{3} \cdot \frac{5+30+55}{3} = 20$$

Review:

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Example:

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30 wins \$4.90

• Why might someone choose \approx 33?

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- Two thirds of the average will be pprox 33
- Any problems?

- Why might someone choose \approx 33?
 - \blacktriangleright If everyone else chooses randomly, the average will be ≈ 50
 - Two thirds of the average will be ≈ 33
- Any problems?
 - If most people think this way, the average will be \approx 33, and so two thirds of the average will be \approx 22

Assume:

Every player is completely rational

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Assume:

- Every player is completely rational
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Are any strategies weakly dominated?

Assume:

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- After crossing these strategies, are any strategies weakly dominated?

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 - Cross these strategies out for everybody
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 - 46 through 67
- ▶ If we continue this process, everyone is left with choosing 1

The class' numbers were: 1, 1, 1, 2.14, 15, 15, 17.9, 18, 30, 32, 34, 34, 37, 45, 45, 48, 48, 53, 80, 89

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Two thirds of the average is 21.53

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- Two thirds of the average is 21.53
- Congratulations Andre Serrano (\$4.80)

Let's play this game again

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- Write down a number between 1 and 100

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- Let's play this game again
- Write down a number between 1 and 100
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What changed?

- Let's play this game again
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4. :

Assume:

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There are two candidates

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- There are two candidates
- ▶ 10% of the voters hold each position

Assume:

- There is a spectrum of 10 points on a certain political issue
- There are two candidates
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- ► Have strategies (1 − 10)

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Is this a game?

- Have players (the candidates)
- ► Have strategies (1 − 10)
- Need payoffs: choose the % of the vote that they earn

Are there any dominated strategies?



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1 is weakly dominated by 2

Are there any dominated strategies?

- 1 is weakly dominated by 2
- 10 is weakly dominated by 9

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Anything else?

Are there any dominated strategies?

- 1 is weakly dominated by 2
- 10 is weakly dominated by 9
- Anything else?
 - 3 does not dominate 2 but after we remove 1 it does (assuming common knowledge)

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If we iterate this, the candidates end up in the central positions

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- Anything else?
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- If we iterate this, the candidates end up in the central positions
- This is The Median Voter Theorem "Majority rule voting will select the median preference"