

# Review

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- ▶ Same goes for the other players
- ▶ There may be other outcomes that are preferable

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  - ▶ All invest, or none invest

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    - ▶ Other players will choose the corresponding equilibrium point

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    - ▶ If you hunt rabbit, the payoff is 1
    - ▶ If you wait for the stag and your partner hunts rabbit, the stag is scared off, and your payoff is 0

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		C	D
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- ▶ Different players prefer different Nash equilibria
- ▶ Basic model for negotiations

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  - ▶ No - only one Nash equilibrium
  - ▶ The best outcome is not a Nash equilibrium

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- ▶ Candidates will split the vote of views that are the same distance to both candidates
- ▶ Win by random draw if candidates tie
- ▶ Payoffs:
  - ▶ Utility of 200 for winning
  - ▶ Cost of 100 to run
  - ▶ Cost of  $|x - y|$  for  $y$  winning (for  $x$ )

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- ▶ Is it a Nash equilibrium if only one person runs?
  - ▶ Only if they lie on 50

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- ▶ Problems?
  - ▶ Everyone decides whether or not to run at once
  - ▶ Not everyone can practically run
  - ▶ Still assumes that politics lie on a single spectrum