The Candidate-Voter Model

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- Payoffs:
  - Utility of 200 for winning
  - Cost of 100 to run
  - Cost of $|x - y|$ for y winning (for x)

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    - No one can opt to run and tie or win

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  - Still assumes that politics lie on a single spectrum

Another Game:

- Consider the following outcome matrix:

	R	P	S
R	0, 0	-1, 1	1, -1
P	1, -1	0, 0	-1, 1
S	-1, 1	1, -1	0, 0

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  - This is an example of a **mixed strategy**

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- What is the expected payout of $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ against $(1, 0, 0)$? $\left(u((\frac{1}{3}, \frac{1}{3}, \frac{1}{3}), (1, 0, 0))\right)$

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- Note that the expected payout is weighted average of the payouts of the pure strategies (with positive probabilities)

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		Date	
		C	D
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	D	0, 0	2, 3

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    - No - you should drop C

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- Note that pure Nash equilibria are still Nash equilibria