Consider Rock Paper Scissors:

	R	Р	S
R	0, 0	-1, 1	1, -1
Р	1, -1	0, 0	-1, 1
\mathbf{S}	-1, 1	1, -1	0, 0

Consider Rock Paper Scissors:

	R	Р	S
R	0, 0	-1, 1	1, -1
Р	1, -1	0, 0	-1, 1
\mathbf{S}	-1, 1	1, -1	0, 0

◆□▶ ◆□▶ ◆三▶ ◆三▶ 三三 のへぐ

• What is the expected payoff of $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ against $(\frac{1}{2}, 0, \frac{1}{2})$?

Consider Rock Paper Scissors:

	R	Р	S
R	0, 0	-1, 1	1, -1
Р	1, -1	0, 0	-1, 1
\mathbf{S}	-1, 1	1, -1	0, 0

◆□▶ ◆□▶ ◆三▶ ◆三▶ 三三 のへぐ

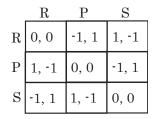
▶ What is the expected payoff of (¹/₃, ¹/₃, ¹/₃) against (¹/₂, 0, ¹/₂)?
 ▶ 0

Consider Rock Paper Scissors:

	R	Р	S
R	0, 0	-1, 1	1, -1
Р	1, -1	0, 0	-1, 1
\mathbf{S}	-1, 1	1, -1	0, 0

- What is the expected payoff of $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ against $(\frac{1}{2}, 0, \frac{1}{2})$?
 - ▶ 0
 - Note that the expected payout is a weighted average of payouts

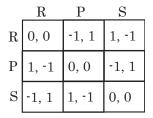
Consider Rock Paper Scissors:



・ロト ・聞ト ・ヨト ・ヨト

æ

Consider Rock Paper Scissors:

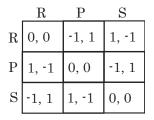


イロト 不得 トイヨト イヨト

3

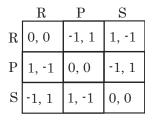
• How can we raise the expected payout against $(\frac{1}{2}, 0, \frac{1}{2})$?

Consider Rock Paper Scissors:



How can we raise the expected payout against (¹/₂, 0, ¹/₂)?
 By removing pure strategies that lower the average

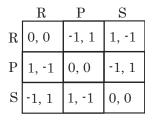
Consider Rock Paper Scissors:



- How can we raise the expected payout against $(\frac{1}{2}, 0, \frac{1}{2})$?
 - By removing pure strategies that lower the average

•
$$u((1,0,0),(\frac{1}{2},0,\frac{1}{2})) = \frac{1}{2}$$

Consider Rock Paper Scissors:

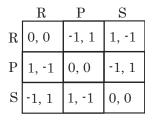


- How can we raise the expected payout against $(\frac{1}{2}, 0, \frac{1}{2})$?
 - By removing pure strategies that lower the average

•
$$u((1,0,0),(\frac{1}{2},0,\frac{1}{2})) = \frac{1}{2}$$

• $u((0,1,0),(\frac{1}{2},0,\frac{1}{2})) = 0$

Consider Rock Paper Scissors:



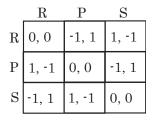
- How can we raise the expected payout against $(\frac{1}{2}, 0, \frac{1}{2})$?
 - By removing pure strategies that lower the average

•
$$u((1,0,0), (\frac{1}{2},0,\frac{1}{2})) = \frac{1}{2}$$

• $u((0,1,0), (\frac{1}{2},0,\frac{1}{2})) = 0$

•
$$u((0,0,1),(\frac{1}{2},0,\frac{1}{2})) = -\frac{1}{2}$$

Consider Rock Paper Scissors:



• How can we raise the expected payout against $(\frac{1}{2}, 0, \frac{1}{2})$?

By removing pure strategies that lower the average

•
$$u((1,0,0), (\frac{1}{2},0,\frac{1}{2})) = \frac{1}{2}$$

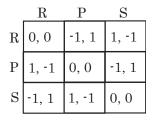
• $u((0,1,0), (\frac{1}{2},0,\frac{1}{2})) = 0$

$$u((0,1,0),(\frac{1}{2},0,\frac{1}{2})) = 0$$

$$u((0,0,1),(\frac{1}{2},0,\frac{1}{2})) = -\frac{1}{2}$$

• Best strategy against $(\frac{1}{2}, 0, \frac{1}{2})$ is

Consider Rock Paper Scissors:



• How can we raise the expected payout against $(\frac{1}{2}, 0, \frac{1}{2})$?

By removing pure strategies that lower the average

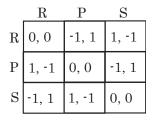
•
$$u((1,0,0), (\frac{1}{2},0,\frac{1}{2})) = \frac{1}{2}$$

•
$$u((0,1,0), (\frac{1}{2},0,\frac{1}{2})) = 0$$

•
$$u((0,0,1),(\frac{1}{2},0,\frac{1}{2})) = -\frac{1}{2}$$

• Best strategy against $(\frac{1}{2}, 0, \frac{1}{2})$ is to play rock

Consider Rock Paper Scissors:



- How can we raise the expected payout against $(\frac{1}{2}, 0, \frac{1}{2})$?
 - By removing pure strategies that lower the average

•
$$u((1,0,0), (\frac{1}{2},0,\frac{1}{2})) = \frac{1}{2}$$

•
$$u((0,1,0),(\frac{1}{2},0,\frac{1}{2})) = 0$$

•
$$u((0,0,1),(\frac{1}{2},0,\frac{1}{2})) = -\frac{1}{2}$$

• Best strategy against $(\frac{1}{2}, 0, \frac{1}{2})$ is to play rock

A best response to a strategy will consist of pure strategies that have the same (high) expected payout

Mixed strategies (p₁,..., p_n) are a Nash equilibrium if p_i is a best response to p_{-i}

(ロ)、(型)、(E)、(E)、 E) の(の)

- ► Mixed strategies (p₁,..., p_n) are a Nash equilibrium if p_i is a best response to p_{-i}
 - Each player asks "if the other players stuck with their strategies, am I better off mixing the ratio of strategies?"

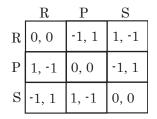
▲□▶ ▲□▶ ▲□▶ ▲□▶ ▲□ ● ● ●

- ► Mixed strategies (p₁,..., p_n) are a Nash equilibrium if p_i is a best response to p_{-i}
 - Each player asks "if the other players stuck with their strategies, am I better off mixing the ratio of strategies?"
 - ► If p_i is a best response to p_{-i}, the payouts of the pure strategies in p_i are equal

- ► Mixed strategies (p₁,..., p_n) are a Nash equilibrium if p_i is a best response to p_{-i}
 - Each player asks "if the other players stuck with their strategies, am I better off mixing the ratio of strategies?"

- ► If p_i is a best response to p_{-i}, the payouts of the pure strategies in p_i are equal
- Note that pure Nash equilibria are still Nash equilibria

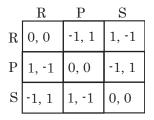
Consider Rock Paper Scissors:



・ロト ・聞ト ・ヨト ・ヨト

æ

Consider Rock Paper Scissors:

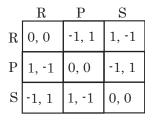


・ロト ・ 雪 ト ・ ヨ ト

æ

What should the (unique) Nash equilibrium be?

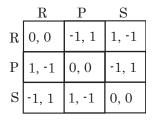
Consider Rock Paper Scissors:



(日) (個) (目) (目) (目) (目)

- What should the (unique) Nash equilibrium be?
 - When both players use $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$

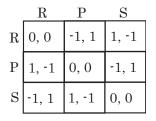
Consider Rock Paper Scissors:



- What should the (unique) Nash equilibrium be?

 - When both players use (¹/₃, ¹/₃, ¹/₃)
 Test this: what is the payoff of a pure strategy against $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)?$

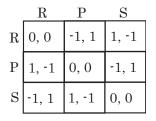
Consider Rock Paper Scissors:



- What should the (unique) Nash equilibrium be?

 - When both players use (¹/₃, ¹/₃, ¹/₃)
 Test this: what is the payoff of a pure strategy against $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)?$

Consider Rock Paper Scissors:



- What should the (unique) Nash equilibrium be?
 - When both players use $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$
 - Test this: what is the payoff of a pure strategy against (¹/₃, ¹/₃, ¹/₃)?
 0
- Note that the expected payoff for each player is 0 (the game is fair)

▶ We saw that there are not always pure Nash equilibrium

◆□ ▶ < 圖 ▶ < 圖 ▶ < 圖 ▶ < 圖 • 의 Q @</p>

▶ We saw that there are not always pure Nash equilibrium

Can we guarantee a mixed Nash equilibrium?

- We saw that there are not always pure Nash equilibrium
- Can we guarantee a mixed Nash equilibrium?

Theorem (Nash)

Suppose that:

- a game has finitely many players
- each player has finitely many pure strategies

we allow for mixed strategies

Then the game admits a Nash equilibrium

> You're playing tennis, and returning the ball

You're playing tennis, and returning the ball

◆□ ▶ < 圖 ▶ < 圖 ▶ < 圖 ▶ < 圖 • 의 Q @</p>

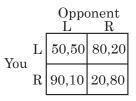
Options:

- You're playing tennis, and returning the ball
- Options:
 - > You can hit the ball to either the opponent's left or right

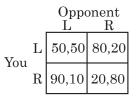
◆□▶ ◆□▶ ◆臣▶ ◆臣▶ 臣 の�?

- You're playing tennis, and returning the ball
- Options:
 - You can hit the ball to either the opponent's left or right
 - Opponent can anticipate where you will hit the ball (to their left or right)

- You're playing tennis, and returning the ball
- Options:
 - You can hit the ball to either the opponent's left or right
 - Opponent can anticipate where you will hit the ball (to their left or right)
 - Payoffs are:

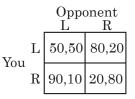


- You're playing tennis, and returning the ball
- Options:
 - You can hit the ball to either the opponent's left or right
 - Opponent can anticipate where you will hit the ball (to their left or right)
 - Payoffs are:



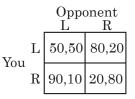
What are the Nash equilibrium?

- You're playing tennis, and returning the ball
- Options:
 - You can hit the ball to either the opponent's left or right
 - Opponent can anticipate where you will hit the ball (to their left or right)
 - Payoffs are:



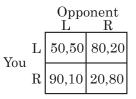
- What are the Nash equilibrium?
 - No pure Nash equilibrium

- You're playing tennis, and returning the ball
- Options:
 - You can hit the ball to either the opponent's left or right
 - Opponent can anticipate where you will hit the ball (to their left or right)
 - Payoffs are:



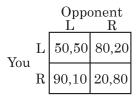
- What are the Nash equilibrium?
 - No pure Nash equilibrium
 - Assume that strategies are (p, 1-p) and (q, 1-q)

- You're playing tennis, and returning the ball
- Options:
 - You can hit the ball to either the opponent's left or right
 - Opponent can anticipate where you will hit the ball (to their left or right)
 - Payoffs are:

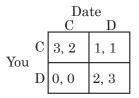


- What are the Nash equilibrium?
 - No pure Nash equilibrium
 - Assume that strategies are (p, 1 p) and (q, 1 q)
 - ► Idea: your opponent's pure strategies must return the same expected payoff (for them) against (p, 1 p)

- You're playing tennis, and returning the ball
- Options:
 - You can hit the ball to either the opponent's left or right
 - Opponent can anticipate where you will hit the ball (to their left or right)
 - Payoffs are:



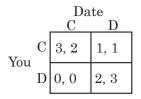
- What are the Nash equilibrium?
 - No pure Nash equilibrium
 - Assume that strategies are (p, 1-p) and (q, 1-q)
 - ► Idea: your opponent's pure strategies must return the same expected payoff (for them) against (p, 1 p)
 - Strategies in Nash equilibrium are (.7, .3)(p = .7) and (.6, .4)(q = .6)



・ロト ・聞ト ・ヨト ・ヨト

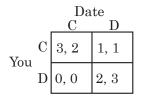
æ

What are the Nash equilibria?

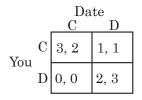


- What are the Nash equilibria?
- Pure Nash equilibria occur when you both go to the same movie

For mixed Nashed equilibria, write out the strategies as (p, 1-p) and (q, 1-q)

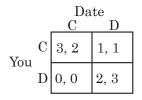


- What are the Nash equilibria?
- Pure Nash equilibria occur when you both go to the same movie
- For mixed Nashed equilibria, write out the strategies as (p, 1-p) and (q, 1-q)
- (Same) trick: to find p, consider your date's pure strategies: the payouts for both strategies must be the same



- What are the Nash equilibria?
- Pure Nash equilibria occur when you both go to the same movie
- For mixed Nashed equilibria, write out the strategies as (p, 1-p) and (q, 1-q)
- (Same) trick: to find p, consider your date's pure strategies: the payouts for both strategies must be the same

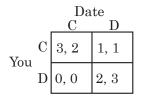
• So
$$2p = p + 3(1 - p)$$



- What are the Nash equilibria?
- Pure Nash equilibria occur when you both go to the same movie
- For mixed Nashed equilibria, write out the strategies as (p, 1-p) and (q, 1-q)
- (Same) trick: to find p, consider your date's pure strategies: the payouts for both strategies must be the same

• So
$$2p = p + 3(1 - p)$$

• $p = \frac{3}{4}$

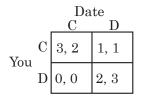


- What are the Nash equilibria?
- Pure Nash equilibria occur when you both go to the same movie
- For mixed Nashed equilibria, write out the strategies as (p, 1-p) and (q, 1-q)
- (Same) trick: to find p, consider your date's pure strategies: the payouts for both strategies must be the same

• So
$$2p = p + 3(1 - p)$$

•
$$p = \frac{1}{4}$$

• Similarly, $q = \frac{1}{4}$

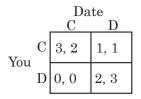


- What are the Nash equilibria?
- Pure Nash equilibria occur when you both go to the same movie
- For mixed Nashed equilibria, write out the strategies as (p, 1-p) and (q, 1-q)
- (Same) trick: to find p, consider your date's pure strategies: the payouts for both strategies must be the same

• So
$$2p = p + 3(1 - p)$$

$$\blacktriangleright p = \frac{3}{4}$$

- Similarly, $q = \frac{1}{4}$
- Note that both you and your date's expected payout is



- What are the Nash equilibria?
- Pure Nash equilibria occur when you both go to the same movie
- For mixed Nashed equilibria, write out the strategies as (p, 1-p) and (q, 1-q)
- (Same) trick: to find p, consider your date's pure strategies: the payouts for both strategies must be the same

• So
$$2p = p + 3(1 - p)$$

•
$$p = \frac{3}{4}$$

- Similarly, $q = \frac{1}{4}$
- Note that both you and your date's expected payout is ³/₂ (between the original payouts of the Nash equilbrium)

Simplification of theory due to John Maynard Smith

(ロ)、(型)、(E)、(E)、 E) の(の)

- Simplification of theory due to John Maynard Smith
- Idea: some small percentage of a population develops a mutation

◆□▶ ◆□▶ ◆臣▶ ◆臣▶ 臣 の�?

- Simplification of theory due to John Maynard Smith
- Idea: some small percentage of a population develops a mutation
- This creates a competing 'strategy', compared to animals without the mutation

◆□▶ ◆□▶ ◆三▶ ◆三▶ 三三 のへぐ

- Simplification of theory due to John Maynard Smith
- Idea: some small percentage of a population develops a mutation
- This creates a competing 'strategy', compared to animals without the mutation

◆□▶ ◆□▶ ◆三▶ ◆三▶ 三三 のへぐ

Will those with the mutation thrive or die?

An example: ants may or may not help defend the nest

- An example: ants may or may not help defend the nest
- This creates a game such as:

Defend Not		
Defend	2, 2	1, 3
Not	3, 1	0, 0

◆□▶ ◆□▶ ◆臣▶ ◆臣▶ 臣 の�?

- An example: ants may or may not help defend the nest
- This creates a game such as:

Suppose that ϵ % (some really small percent) of the ants have the mutation, and $100 - \epsilon$ % don't

- An example: ants may or may not help defend the nest
- This creates a game such as:

Suppose that ϵ % (some really small percent) of the ants have the mutation, and $100 - \epsilon$ % don't

• Payoff for a mutant is bigger when ϵ is small

- An example: ants may or may not help defend the nest
- This creates a game such as:

- Suppose that ϵ % (some really small percent) of the ants have the mutation, and 100ϵ % don't
- Payoff for a mutant is bigger when ϵ is small
- Percent of population with mutation will grow until Nash equilibrium is reached