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- If $(s,s)$ is a Nash equilibrium, $s$ is not necessarily evolutionarily stable

# Definition

Another definition for evolutionarily stable strategies:
In a 2-player symmetric game, a strategy $s$ is **evolutionarily stable** if:

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- This definition is far easier to check

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- $p$ is a mixed evolutionarily stable strategy

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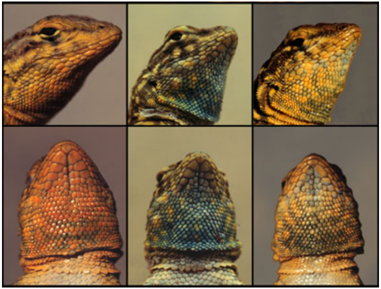

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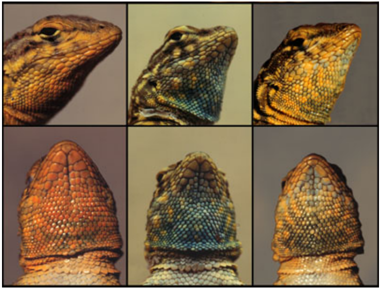

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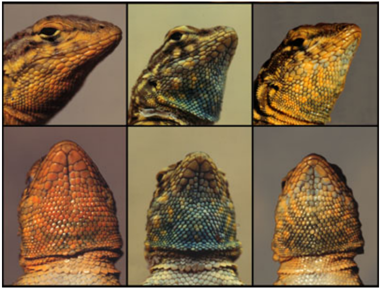

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- Males have three possible colorings (orange-blue-yellow)
- Colorings corresponding to mating habits

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- Only evolutionarily stable strategy is Orange

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