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 - ▶ House in Manhattan, Kansas

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 - ▶ House in Manhattan, Kansas
 - ▶ Sailboat

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- ▶ Note that previous methods don't work well
- ▶ Instead, have heirs bid on items

Method of Sealed Bids

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4. **Division of Surplus:** remaining money is divided equally among the parties

Method of Sealed Bids

Bidding: each player determines how much each item is worth, then submits a bid on each item (in 1000s)

	Albert	Beatrice	Clyde	Diane
Tokyo				
Manhattan				
Boat				
Painting				

Method of Sealed Bids

Bidding: each player determines how much each item is worth, then submits a bid on each item (in 1000s)

	Albert	Beatrice	Clyde	Diane
Tokyo	400	500	360	420
Manhattan	30	50	60	40
Boat	50	30	40	40
Painting	6	20	4	10

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Item Value	50	520	60	0
Settlement	71.5	-370	56	127.5

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 - ▶ Beatrice gets the Tokyo house, the painting, and has to pay the estate \$370,000
 - ▶ Clyde gets the Manhattan house, and receives \$56,000

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 - ▶ Diane receives \$127,500

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 - ▶ Diane receives \$127,500
- ▶ Are we done?

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- ▶ Are we done?
 - ▶ No - there is \$115,000 in the estate

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 - ▶ Beatrice gets the Tokyo house, the painting, and has to pay the estate \$370,000
 - ▶ Clyde gets the Manhattan house, and receives \$56,000
 - ▶ Diane receives \$127,500
- ▶ Are we done?
 - ▶ No - there is \$115,000 in the estate
 - ▶ Split this equally

Method of Sealed Bids

Division of Surplus: remaining money is divided equally among the parties

	Albert	Beatrice	Clyde	Diane
Tokyo	400	500	360	420
Manhattan	30	50	60	40
Boat	50	30	40	40
Painting	6	20	4	10
Total	486	600	464	510
Fair Share	121.5	150	116	127.5
Item Value	50	520	60	0
Settlement	71.5	-370	56	127.5
Surplus	28.75	28.75	28.75	28.75

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Tokyo	400	500	360	420
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Painting	6	20	4	10
Total	486	600	464	510
Fair Share	121.5	150	116	127.5
Item Value	50	520	60	0
Settlement	71.5	-370	56	127.5
Surplus	28.75	28.75	28.75	28.75
Final Settlement	100.25	-341.25	84.75	156.25

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 - ▶ Auction theory says expected values would equal (with open bids)
 - ▶ If bids were open, would want to just undercut highest bidder:

Open Bidding

Suppose bids were open:

	Albert	Beatrice	Clyde	Diane
Tokyo	499	500	499	499
Manhattan	59	59	60	59
Boat	50	49	49	49
Painting	19	20	19	19
Total	627	628	627	626

Open Bidding

Suppose bids were open:

	Albert	Beatrice	Clyde	Diane
Tokyo	499	500	499	499
Manhattan	59	59	60	59
Boat	50	49	49	49
Painting	19	20	19	19
Total	627	628	627	626
Fair Share	156.75	157	156.75	156.5

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Tokyo	499	500	499	499
Manhattan	59	59	60	59
Boat	50	49	49	49
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Total	627	628	627	626
Fair Share	156.75	157	156.75	156.5
Item Value	50	520	60	0
Settlement				

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	Albert	Beatrice	Clyde	Diane
Tokyo	499	500	499	499
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Painting	19	20	19	19
Total	627	628	627	626
Fair Share	156.75	157	156.75	156.5
Item Value	50	520	60	0
Settlement	106.75	-363	96.75	156.5

Open Bidding

Suppose bids were open:

	Albert	Beatrice	Clyde	Diane
Tokyo	499	500	499	499
Manhattan	59	59	60	59
Boat	50	49	49	49
Painting	19	20	19	19
Total	627	628	627	626
Fair Share	156.75	157	156.75	156.5
Item Value	50	520	60	0
Settlement	106.75	-363	96.75	156.75
Surplus	.75	.75	.75	.75

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Fair Share	156.75	157	156.75	156.5
Item Value	50	520	60	0
Settlement	106.75	-363	96.75	156.75
Surplus	.75	.75	.75	.75
Final Settlement	107.5	-362.25	97.5	157.5

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	Albert	Beatrice	Clyde	Diane
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Total	627	628	627	626
Fair Share	156.75	157	156.75	156.5
Item Value	50	520	60	0
Settlement	106.75	-363	96.75	156.75
Surplus	.75	.75	.75	.75
Final Settlement	107.5	-362.25	97.5	157.5

Open bidding encourages false bids (can get more \$\$)

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- ▶ Problems?

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 - ▶ Encourages honesty
 - ▶ If you underbid, your “fair share” decreases
 - ▶ If you overbid, you’ll pay a lot of money for something you don’t want
- ▶ Problems?
 - ▶ Requires players to have enough cash to cover bids

Method of Sealed Bids

- ▶ See Handout #7