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  - Everybody has individual preferences
  - Want to transform individual preferences to a single societal preference
  - Want to do this fairly

Plurality Voting

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    - Example of vote splitting

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  - 2000 Presidential election:
    - Bush: 48.38%
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    - Probable that many preferred Nader, but did not want to "throw away their vote"

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- Used in French presidential elections

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- Problems:
  - Inefficient; need to hold election over multiple days

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Voter 3		✓	✓
Voter 4	✓	✓	

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- Used in many professional societies, and the election for the U.N. Secretary-General

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- Problems will be covered later

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Most Preferred	A	B	B	B	C	C	A	C	B
	B	C	C	C	A	A	C	B	C
Least Preferred	C	A	A	A	B	B	B	A	A

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	Voters								
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	B	C	C	C	A	A	C	B	C
Least Preferred	C	A	A	A	B	B	B	A	A

- Question: how do we tally the votes?

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- Back to the example:

	Voters								
Most Preferred	*A*	*B*	*B*	*B*	*C*	*C*	*A*	*C*	*B*
	B	*C*	*C*	*C*	*A*	*A*	*C*	*B*	*C*
Least Preferred	*C*	*A*	*A*	*A*	*B*	*B*	*B*	*A*	*A*

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Most Preferred	*A*	*B*	*B*	*B*	*C*	*C*	*A*	*C*	*B*
	B	*C*	*C*	*C*	*A*	*A*	*C*	*B*	*C*
Least Preferred	*C*	*A*	*A*	*A*	*B*	*B*	*B*	*A*	*A*

- *A* has 2 votes
- *B* has 4 votes
- *C* has 3 votes

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Most Preferred	~~*A*~~	*B*	*B*	*B*	*C*	*C*	~~*A*~~	*C*	*B*
	B	*C*	*C*	*C*	~~*A*~~	~~*A*~~	*C*	*B*	*C*
Least Preferred	*C*	~~*A*~~	~~*A*~~	~~*A*~~	*B*	*B*	*B*	~~*A*~~	~~*A*~~

- ~~*A* has 2 votes~~

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	B	*C*	*C*	*C*	~~*A*~~	~~*A*~~	*C*	*B*	*C*
Least Preferred	*C*	~~*A*~~	~~*A*~~	~~*A*~~	*B*	*B*	*B*	~~*A*~~	~~*A*~~

- ~~*A* has 2 votes~~
- *B* has 5 votes
- *C* has 4 votes

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	B	C	C	C	~~A~~	~~A~~	C	B	C
Least Preferred	C	~~A~~	~~A~~	~~A~~	B	B	B	~~A~~	~~A~~

- ~~A has 2 votes~~
- B has 5 votes
- C has 4 votes

So B wins

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- A gets $3 \cdot 2 + 2 \cdot 2 + 1 \cdot 5 = 15$ points

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- B gets $3 \cdot 4 + 2 \cdot 2 + 1 \cdot 3 = 19$ points

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- One good idea is the **Condorcet criterion**:
  - A candidate is the **Condorcet winner** if they would win in head-to-head competition with any other candidate
  - A voting method satisfies the **Condorcet criterion** if a Condorcet winner will always win

Plurality Voting and the Condorcet Criterion

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  - 32% prefer A then B then C
  - 28% prefer B then A then C
  - 40% prefer C then A then B

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  - A would get 72%; B would get 36%

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- In a plurality election, C wins the election!
- Plurality voting does not satisfy the Condorcet criterion