Review

Methods of Voting:

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Number of Voters	**14**	**10**	**8**	**4**	**1**
1^{st} **choice**	A	C	D	B	C
2^{nd} **choice**	B	B	C	D	D
3^{rd} **choice**	C	D	B	C	B
4^{th} **choice**	D	A	A	A	A

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- Who is the instant runoff winner?
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- Who is the instant runoff winner?
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- Who is the Borda count winner?

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- Who should win the election?

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- Who is the plurality winner?
  - A
- Who is the instant runoff winner?
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- Who is the Borda count winner?
  - B
- Who should win the election?
  - Need to consider which voting methods are fair

Notions of Fairness

Condorcet Criterion:

- A candidate is the **Condorcet winner** if they would win in head-to-head competition with any other candidate

Notions of Fairness

Condorcet Criterion:

- A candidate is the **Condorcet winner** if they would win in head-to-head competition with any other candidate
- A voting method satisfies the **Condorcet criterion** if a Condorcet winner will always win the election

Condorcet Criterion

Number of Voters	**11**	**10**	**8**
1^{st} **choice**	A	B	C
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Condorcet Criterion

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- Who is the Condorcet winner?
  - There isn't one!

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- Question: is this realistic?

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  - There isn't one!
- Question: is this realistic?
  - Could have:
    - A is economically progressive and socially liberal
    - B is fiscally conservative and socially liberal
    - C is fiscally conservative and socially conservative

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- So plurality voting, instant runoffs, and the Borda count don't satisfy the Condorcet criterion

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- So plurality voting, instant runoffs, and the Borda count don't satisfy the Condorcet criterion
- Two questions:
  - Is there a voting method that satisfies the Condorcet criterion?
  - What are other notions of fair?

Other Notions of Fairness

- Majority Criterion:

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- Does plurality voting satisfy the majority criterion?
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  - Yes
- Does the Borda method satisfy the majority criterion?

Borda Method and the Majority Criterion

Number of Voters	**11**	**5**	**5**
1^{st} **choice**	A	B	C
2^{nd} **choice**	B	C	B
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Borda Method and the Majority Criterion

Number of Voters	**11**	**5**	**5**
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  - A
- Who is the Borda winner?
  - B
- So the Borda method does not satisfy the majority criterion

Summary

	Condorcet	Majority
Plurality	no	yes
Instant Runoff	no	yes
Borda	no	no

Other Notions of Fairness

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  - Yes - a public enemy cannot get enough points

Summary

	Condorcet	Majority	Public Enemy
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Instant Runoff	no	yes	yes
Borda	no	no	yes

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- Who wins?
  - Madrid is eliminated in the first round
  - Rio de Janeiro wins!?!
- So Tokyo getting more first round votes caused them to lose