

Review

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- ▶ Condorcet criterion:
 - ▶ A candidate is the **Condorcet winner** if they would win in head-to-head competition with any other candidate
 - ▶ A voting method satisfies the **Condorcet criterion** if a Condorcet winner will always win the election
- ▶ majority criterion:
 - ▶ A voting method satisfies the **majority criterion** if a candidate with a majority of first-preference votes will win the election

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Fairness criteria for voting methods:

- ▶ Condorcet criterion:
 - ▶ A candidate is the **Condorcet winner** if they would win in head-to-head competition with any other candidate
 - ▶ A voting method satisfies the **Condorcet criterion** if a Condorcet winner will always win the election
- ▶ majority criterion:
 - ▶ A voting method satisfies the **majority criterion** if a candidate with a majority of first-preference votes will win the election
- ▶ public enemy criterion:
 - ▶ A voting method satisfies the **public enemy criterion** is a candidate with a majority of last-preference votes **cannot** win the election

Example

Number of Voters	7	5	6
1st choice	<i>A</i>	<i>B</i>	<i>C</i>
2nd choice	<i>B</i>	<i>C</i>	<i>B</i>
3rd choice	<i>C</i>	<i>A</i>	<i>A</i>

- ▶ Is there a Condorcet winner?

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- ▶ Is there a Condorcet winner?
 - ▶ *B*
- ▶ Is there a majority winner?
 - ▶ no

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<i>3rd</i> choice	<i>C</i>	<i>A</i>	<i>A</i>

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- ▶ Is there a Condorcet winner?
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1st choice	<i>A</i>	<i>B</i>	<i>C</i>
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- ▶ Is there a Condorcet winner?
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- ▶ Who is the instant runoff winner (10 to win)?

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- ▶ Is there a Condorcet winner?
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- ▶ Is there a majority winner?
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- ▶ Is there a public enemy?
 - ▶ A
- ▶ Who is the plurality winner?
 - ▶ A
- ▶ Who is the instant runoff winner (10 to win)?
 - ▶ C
- ▶ Is there Borda winner?

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Number of Voters	7	5	6
1st choice	<i>A</i>	<i>B</i>	<i>C</i>
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- ▶ Is there a Condorcet winner?
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- ▶ Who is the plurality winner?
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- ▶ Who is the instant runoff winner (10 to win)?
 - ▶ *C*
- ▶ Is there Borda winner?
 - ▶ *B*

Summary

	Condorcet	Majority	Public Enemy
Plurality	no	yes	no
Instant Runoff	no	yes	yes
Borda	no	no	yes

Example

- ▶ Instant runoffs are being used to determine the host city for the 2016 Olympics

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Number of Voters	7	8	10	4
<i>1st</i> choice	<i>M</i>	<i>R</i>	<i>T</i>	<i>M</i>
<i>2nd</i> choice	<i>R</i>	<i>T</i>	<i>M</i>	<i>T</i>
<i>3rd</i> choice	<i>T</i>	<i>M</i>	<i>R</i>	<i>R</i>

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<i>3rd</i> choice	<i>T</i>	<i>M</i>	<i>R</i>	<i>R</i>

- ▶ If the election were held right now, who would win?

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- ▶ If the election were held right now, who would win?
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- ▶ If the election were held right now, who would win?
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3rd choice	<i>T</i>	<i>M</i>	<i>R</i>	<i>R</i>

- ▶ The change looks like it only benefits Tokyo (the presumed winner)
- ▶ Who wins?
 - ▶ Madrid is eliminated in the first round
 - ▶ Rio de Janeiro wins!?!
- ▶ So Tokyo getting more first round votes caused them to lose

The Monotonicity Criterion

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- ▶ Does plurality voting satisfy the monotonicity criterion?

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- ▶ Does plurality voting satisfy the monotonicity criterion?
 - ▶ Yes

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- ▶ Instant runoffs do not satisfy the monotonicity criterion
- ▶ Does plurality voting satisfy the monotonicity criterion?
 - ▶ Yes
- ▶ Does the Borda method satisfy the monotonicity criterion?

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- ▶ The monotonicity criterion:
 - ▶ A voting method satisfies the **monotonicity criterion** if the winner of an election cannot lose a repeat election if preferences are altered **only** to the benefit of the original winner
- ▶ Instant runoffs do not satisfy the monotonicity criterion
- ▶ Does plurality voting satisfy the monotonicity criterion?
 - ▶ Yes
- ▶ Does the Borda method satisfy the monotonicity criterion?
 - ▶ Yes

Summary

	Condorcet	Majority	Public Enemy	Monotonicity
Plurality	no	yes	no	yes
Instant Runoff	no	yes	yes	no
Borda	no	no	yes	yes

One Last Criterion

Number of Voters	8	6	4	3
<i>1st</i> choice	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>
<i>2nd</i> choice	<i>D</i>	<i>C</i>	<i>D</i>	<i>B</i>
<i>3rd</i> choice	<i>B</i>	<i>A</i>	<i>A</i>	<i>C</i>
<i>4th</i> choice	<i>C</i>	<i>D</i>	<i>B</i>	<i>A</i>

- ▶ Who is the plurality winner?

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Number of Voters	8	6	4	3
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<i>2nd</i> choice	<i>D</i>	<i>C</i>	<i>D</i>	<i>B</i>
<i>3rd</i> choice	<i>B</i>	<i>A</i>	<i>A</i>	<i>C</i>
<i>4th</i> choice	<i>C</i>	<i>D</i>	<i>B</i>	<i>A</i>

- ▶ Who is the plurality winner?
 - ▶ A

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<i>2nd</i> choice	<i>D</i>	<i>C</i>	<i>D</i>	<i>B</i>
<i>3rd</i> choice	<i>B</i>	<i>A</i>	<i>A</i>	<i>C</i>
<i>4th</i> choice	<i>C</i>	<i>D</i>	<i>B</i>	<i>A</i>

- ▶ Who is the plurality winner?
 - ▶ *A*
- ▶ Who is the instant runoff winner (11 to win)?

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Number of Voters	8	6	4	3
<i>1st</i> choice	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>
<i>2nd</i> choice	<i>D</i>	<i>C</i>	<i>D</i>	<i>B</i>
<i>3rd</i> choice	<i>B</i>	<i>A</i>	<i>A</i>	<i>C</i>
<i>4th</i> choice	<i>C</i>	<i>D</i>	<i>B</i>	<i>A</i>

- ▶ Who is the plurality winner?
 - ▶ A
- ▶ Who is the instant runoff winner (11 to win)?
 - ▶ A

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Number of Voters	8	6	4	3
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<i>2nd</i> choice	<i>D</i>	<i>C</i>	<i>D</i>	<i>B</i>
<i>3rd</i> choice	<i>B</i>	<i>A</i>	<i>A</i>	<i>C</i>
<i>4th</i> choice	<i>C</i>	<i>D</i>	<i>B</i>	<i>A</i>

- ▶ Who is the plurality winner?
 - ▶ *A*
- ▶ Who is the instant runoff winner (11 to win)?
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- ▶ Who is the Borda winner?

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<i>1st</i> choice	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>
<i>2nd</i> choice	<i>D</i>	<i>C</i>	<i>D</i>	<i>B</i>
<i>3rd</i> choice	<i>B</i>	<i>A</i>	<i>A</i>	<i>C</i>
<i>4th</i> choice	<i>C</i>	<i>D</i>	<i>B</i>	<i>A</i>

- ▶ Who is the plurality winner?
 - ▶ *A*
- ▶ Who is the instant runoff winner (11 to win)?
 - ▶ *A*
- ▶ Who is the Borda winner?
 - ▶ *A*

One Last Criterion

Now suppose that B drops out:

Number of Voters	8	6	4	3
1 st choice	A	B C	C	D
2 nd choice	D	X A	D	B C
3 rd choice	B C	X D	A	X A
	X	D	B	X

- ▶ Who is the plurality winner?

One Last Criterion

Now suppose that B drops out:

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1 st choice	A	B C	C	D
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3 rd choice	B C	X D	A	X A
	X	D	B	X

- ▶ Who is the plurality winner?
 - ▶ C

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Now suppose that B drops out:

Number of Voters	8	6	4	3
1 st choice	A	B C	C	D
2 nd choice	D	X A	D	B C
3 rd choice	B C	X D	A	X A
	X	D	B	X

- ▶ Who is the plurality winner?
 - ▶ C
- ▶ Who is the instant runoff winner (11 to win)?

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Now suppose that B drops out:

Number of Voters	8	6	4	3
1 st choice	A	B C	C	D
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3 rd choice	B C	X D	A	X A
	X	D	B	X

- ▶ Who is the plurality winner?
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- ▶ Who is the instant runoff winner (11 to win)?
 - ▶ C

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Now suppose that B drops out:

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1 st choice	A	B C	C	D
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	X	D	B	X

- ▶ Who is the plurality winner?
 - ▶ C
- ▶ Who is the instant runoff winner (11 to win)?
 - ▶ C
- ▶ Who is the Borda winner?

One Last Criterion

Now suppose that B drops out:

Number of Voters	8	6	4	3
1 st choice	A	B C	C	D
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	X	D	B	X

- ▶ Who is the plurality winner?
 - ▶ C
- ▶ Who is the instant runoff winner (11 to win)?
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- ▶ Who is the Borda winner?
 - ▶ C

Independence of Irrelevant Alternatives

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- ▶ The independence of irrelevant alternatives criterion:
 - ▶ A voting method satisfies the **I.I.A. criterion** if the winner of an election would still win if other candidates were disqualified
- ▶ Plurality, instant runoffs, and the Borda method do not satisfy I.I.A.

Summary

	Cond.	Maj.	P.E.	Mono.	I.I.A.
Plurality	no	yes	no	yes	no
Instant Runoff	no	yes	yes	no	no
Borda	no	no	yes	yes	no

Method of Pairwise Comparisons (Condorcet Method)

Method of pairwise comparisons

- ▶ Compare each pair of candidates

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Method of pairwise comparisons

- ▶ Compare each pair of candidates
- ▶ Candidate earns one point for each candidate that they beat
- ▶ Candidate with the most points wins

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Number of Voters	7	5	6
1st choice	<i>A</i>	<i>B</i>	<i>C</i>
2nd choice	<i>B</i>	<i>C</i>	<i>B</i>
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- ▶ Who wins via the method of pairwise comparisons?

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- ▶ Who wins via the method of pairwise comparisons?
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<i>3rd</i> choice	<i>C</i>	<i>A</i>	<i>A</i>

- ▶ Who wins via the method of pairwise comparisons?
 - ▶ *B*
- ▶ The method of pairwise comparisons satisfies the Condorcet criterion

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3rd choice	<i>C</i>	<i>A</i>	<i>A</i>

- ▶ Who wins via the method of pairwise comparisons?
 - ▶ *B*
- ▶ The method of pairwise comparisons satisfies the Condorcet criterion
- ▶ Does it satisfy the majority criterion?

Example

Number of Voters	7	5	6
1st choice	<i>A</i>	<i>B</i>	<i>C</i>
2nd choice	<i>B</i>	<i>C</i>	<i>B</i>
3rd choice	<i>C</i>	<i>A</i>	<i>A</i>

- ▶ Who wins via the method of pairwise comparisons?
 - ▶ *B*
- ▶ The method of pairwise comparisons satisfies the Condorcet criterion
- ▶ Does it satisfy the majority criterion?
 - ▶ yes

Example

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2nd choice	<i>B</i>	<i>C</i>	<i>B</i>
3rd choice	<i>C</i>	<i>A</i>	<i>A</i>

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1st choice	<i>A</i>	<i>B</i>	<i>C</i>
2nd choice	<i>B</i>	<i>C</i>	<i>B</i>
3rd choice	<i>C</i>	<i>A</i>	<i>A</i>

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Example

Number of Voters	7	5	6
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- ▶ Does it satisfy the I.I.A. criterion?

Example

Number of Voters	7	5	6
1st choice	<i>A</i>	<i>B</i>	<i>C</i>
2nd choice	<i>B</i>	<i>C</i>	<i>B</i>
3rd choice	<i>C</i>	<i>A</i>	<i>A</i>

- ▶ Who wins via the method of pairwise comparisons?
 - ▶ *B*
- ▶ The method of pairwise comparisons satisfies the Condorcet criterion
- ▶ Does it satisfy the majority criterion?
 - ▶ yes
- ▶ Does it satisfy the public enemy criterion?
 - ▶ yes
- ▶ Does it satisfy the monotonicity criterion?
 - ▶ yes
- ▶ Does it satisfy the I.I.A. criterion?
 - ▶ no

Method of Pairwise Comparisons

Problems with the method of pairwise comparisons?

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3rd choice	C	A	B

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Number of Voters	7	5	6
1st choice	A	B	C
2nd choice	B	C	A
3rd choice	C	A	B

- ▶ Who wins via the method of pairwise comparisons?
 - ▶ Nobody (rock, paper, scissors)

Summary

	Cond.	Maj.	P.E.	Mono.	I.I.A.
Plurality	no	yes	no	yes	no
Instant Runoff	no	yes	yes	no	no
Borda	no	no	yes	yes	no
Pairwise Comparisons	yes	yes	yes	yes	no

Another Method?

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Arrow's Impossibility Theorem

There is no voting method that satisfies:

- ▶ Condorcet criterion
- ▶ majority criterion
- ▶ monotonicity criterion
- ▶ I.I.A. criterion