New Due Dates

- Partial rough draft: Monday, December 2
- Final paper: Wednesday, December 11

Tie Breaking

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- If voters are perfectly rational, who will win?

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- After eliminating strategies, Voter 3 will opt to vote for C (voting for C weakly dominates voting for A)
- After eliminating strategies, Voter 2 will vote for C (voting for C weakly dominates voting for B)
- Winner is C
- This is referred to as the **Chair's Paradox**

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  - Shareholders' meetings
    - Shareholder's vote is weighted by their number of shares

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  - Permanent members have veto power
  - How can we quantify the difference in power?

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- Notation is $[q : w_1, \ldots, w_n]$

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  - A, B, and C all have the same power
    - Decision goes to which ever two agree

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- Consider $[10 : 11, 3, 3, 3]$

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  - Motion needs 10 votes to pass
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  - Motion needs 10 votes to pass
  - A gets 11 votes; B gets 3 votes; C gets 3 votes; D gets 3 vote
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  - A decides the outcome
    - A is a **dictator**
      (their vote determines the outcome)

Example

- Consider $[10 : 11, 3, 3, 3]$
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- Note:
  - A decides the outcome
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  - B, C, and D are necessarily dummy votes

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  - Motion needs 16 votes to pass
  - A gets 8 votes; B gets 7 votes; C gets 3 votes; D gets 2 vote
- Note:
  - A is not a dictator

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  - Motion needs 16 votes to pass
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  - A is not a dictator
  - However, A has **veto power**

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  - B also has veto power
  - C and D are **not** dummy votes

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- The Shapley-Shubik power index of a voter is the ratio of how often they are pivotal to the total number of orderings
- For n voters, there are $n! = n \cdot (n-1) \cdot \ldots \cdot 1$ orderings

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A C B	C
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B C A	A
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- B's index is $\frac{1}{6}$
- C's index is $\frac{1}{6}$
- B and C have the same power

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- Dictator's have index 1

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- Dictator's have index 1
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- Maryland's (10) index is 1.82%
- Pennsylvania's (20) index is 3.91%
- Texas' (38) index is 6.50%