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- Total number of votes is $V = w_1 + w_2 + \ldots + w_N$
- There is a quota q
  - Number of votes needed to pass a motion
- Notation for a weighted voting system is $[q : w_1, \ldots, w_N]$

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- The set of all voters is called **the grand coalition**

Example

- Consider $[58; 30, 30, 25, 20, 5, 1]$

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Enumeration

Question: How many different possible coalitions are there?

- $N = 1$:

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  - There are 2^N options for whether or not N voters are in a coalition
  - The empty set is not a coalition
  - There are $2^N - 1$ coalitions

Example

Consider $[3; 2, 1, 1]$

Example

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Coalition	# of Votes	Winning/Losing
$\{P_1\}$	2	
$\{P_2\}$	1	
$\{P_3\}$	1	
$\{P_1, P_2\}$	3	
$\{P_1, P_3\}$	3	
$\{P_2, P_3\}$	2	
$\{P_1, P_2, P_3\}$	5	

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$\{P_1\}$	2	Losing
$\{P_2\}$	1	Losing
$\{P_3\}$	1	Losing
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$\{P_1, P_3\}$	3	Winning
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$\{P_2, P_3\}$	2	Losing
$\{P_1, P_2, P_3\}$	5	Winning

Note that P_1 has veto power (it is in every winning coalition)

Examples

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Want to consider how power is distributed among the three voters

	[10;5,5,4]		[60;44,22,11]	
Coalition	**# of Votes**	**W/L**	**# of Votes**	**W/L**
$\{P_1\}$	5			
$\{P_2\}$	5			
$\{P_3\}$	4			
$\{P_1, P_2\}$	10			
$\{P_1, P_3\}$	9			
$\{P_2, P_3\}$	9			
$\{P_1, P_2, P_3\}$	14			

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$\{P_1\}$	5	Losing	44	
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$\{P_3\}$	4	Losing	11	
$\{P_1, P_2\}$	10	Winning	66	
$\{P_1, P_3\}$	9	Losing	55	
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The weighted voting systems have the same winning coalitions
These weighted voting systems are said to be **equivalent**

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- How can we tell if P_1 is a dictator?

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Coalition	# of Votes	Winning/Losing
$\{P_1\}$	5	
$\{P_2\}$	2	
$\{P_3\}$	2	
$\{P_1, P_2\}$	7	
$\{P_1, P_3\}$	7	
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P_1 is a dictator if $\{P_1\}$ is a winning strategy
P_N is a dummy voter if their removal from any winning coalition is still a winning coalition

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$\{P_1\}$	7		
$\{P_2\}$	5		
$\{P_3\}$	4		
$\{P_1, P_2\}$	12		
$\{P_1, P_3\}$	11		
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$\{P_1, P_2, P_3\}$	16		

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- The Banzhaf Index of P_i is

$$\beta(P_i) = \frac{\#\text{ of times } P_i \text{ is crit.}}{\#\text{ of crit. members over all coalitions}}$$

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  - 5 permanent members:
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  - 10 non-permanent members:
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  - Each member has one vote
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  - Each member has one vote
  - Require nine votes to pass a motion
  - Permanent members have veto power

U.N. Security Council

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  - 84 such coalitions

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- Permanent members have Banzhaf Index

$$\frac{848}{5 \cdot 848 + 10 \cdot 84} = \frac{848}{5080} \approx 16.69\%$$

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$$\frac{848}{5 \cdot 848 + 10 \cdot 84} = \frac{848}{5080} \approx 16.69\%$$

- Non-permanent members have Banzhaf Index

$$\frac{84}{5080} \approx 1.65\%$$

Electoral Votes

Electoral votes of certain states:

Electoral Votes

Electoral votes of certain states:

State	# of E.V.	**% of E.V.**	**SS Index**	**B Index**
Nevada	5	0.93%	0.93%	0.90%
Maryland	10	1.86%	1.86%	1.82%
Pennsylvania	20	3.72%	3.72%	3.91%
Texas	38	7.06%	6.39%	6.50%