Administration

- New due dates:
  - Homework #10 is available now, and will be due on Monday, December 9.
  - Final paper is due Wednesday, December 11.
  - Homework #11 will be due Friday, December 13.

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    - This is an example of an **exponential function** (these look like $p \cdot a^t$ for constants p,a)

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  - If you invest $\$P$ with $k\%$ interest, compounded continuously, the investment is worth $P \cdot e^{kt}$

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  - Problem: the bank is unwilling to lend you money

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  - This is called a **future**

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- Suppose that the rate **increases** to €1 = \$1.05
  - Euros are worth \$525

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  - Owes bank \$475

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  - You don't exercise your option

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  - $0 = 0.95x - 1.01y$
- Solve to get $x = 500$, $y = 470.30$
- The trader is out \$29.70
  - The price they charged you

For Next Time:

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- What happens if we have multiple Bernoulli trials?