

Math 584, Problem set 6 due December 7, 2006

Dr. Epstein

Reading: Read Chapters 9.1–9.1.6, 9.2, 10.1–10.3 and 10.5, 11.1–11.4. A final matlab worksheet will also be assigned. Prepare your class presentation. The written version should be handed in no later than December 8.

1. From the text do problems: 8.1.4, 8.2.8, 8.2.12, (here assume that $\widehat{\varphi}(0) = 1$), 8.3.3, 8.3.4
2. Let $f(x) = \frac{1 - \cos(2x)}{\pi x^2}$.
 - (a) Show that this function is bandlimited and determine the Nyquist sampling rate for it.
 - (b) Suppose that we sample at half the Nyquist rate and apply the Shannon-Whittaker formula to obtain a function $F(x)$ which interpolates the samples and whose Fourier transform is supported in a band of half the length. What is $\widehat{F}(\xi)$?
 - (c) Graph $f(x)$ and $F(x)$ on the same plot.
3. Suppose that f is an L -bandlimited function and g is an M -bandlimited function, show that $fg(x)$ is an $L + M$ -bandlimited function.
4. Edge detection is an important operation in image processing. In this problem we consider a 1-dimensional analogue. Suppose that $f(x)$ is the density of an image at the point x . We can assume that $0 \leq f(x) \leq 1$. Find a mathematical description for an “edge.” Explain why edge detection is a shift invariant operation. Design a practical filter for detecting edges and implement it using an FFT.
5. Do exercises 9.2.5, 9.2.7 from the book.
6. Do exercises 10.2.3, 10.2.4 from the book.

7. Recall Simpson's rule: if $f_j = f(a + jd)$, where $d = (b - a)/N$, (N an even number), then

$$\int_a^b f(x) \approx \frac{d}{3} [f_0 + 4f_1 + 2f_2 + 4f_3 + \dots + 2f_{N-2} + 4f_{N-1} + f_N].$$

By modifying the definition of $\langle f_s(j) \rangle$ we can arrange for

$$(1) \quad \frac{2N-1}{2N} \mathcal{F}_{2N-1}^{-1}(\langle \hat{h}_s(0)\hat{f}_s(0), \dots, \hat{h}_s(2N-2)\hat{f}_s(2N-2) \rangle)(k)$$

to give a Simpson's rule approximation for the samples of the convolution:

$$\int_0^1 h\left(\frac{k}{N} - y\right) f(y) dy.$$

Explain how the definition of $\langle f_s(j) \rangle$ should be modified to achieve this.