

Math 241, Exam 1

May 29, 2007

Name: _____

Instructor: **Peter Dalakov**

In order to receive full credit you need to show all your work .

<i>Score</i>		
1	4	
2	4	
3	4	
4	4	
5	4	
<i>Total</i>	20	

Convenient formulas and notation:

- We can write $z \in \mathbb{C}$ as

$$z = x + iy = |z|(\cos \theta + i \sin \theta) = re^{i\theta}, \quad r = |z| = \sqrt{x^2 + y^2}$$

$$x = \operatorname{Re}(z) = \frac{z + \bar{z}}{2}, \quad y = \operatorname{Im}(z) = \frac{z - \bar{z}}{2i}, \quad \bar{z} := x - iy$$

- Euler's formula:

$$e^{i\theta} = \cos \theta + i \sin \theta$$

- The Cauchy-Riemann equations for $f = u + iv$ are

$$u_x = v_y, \quad u_y = -v_x$$

1. Find and draw all $z \in \mathbb{C}$, such that

$$z^3 + i = 0$$

Draw the unit circle on the same picture. Make the picture **big**.

Hint: **Do not** quote from memory de Moivre's formula, but just write what does the equation say for $z = re^{i\theta}$.

2. Find a function u such that $f = u + iv$ is analytic in \mathbb{C} and

$$v(x, y) = x^2 - y^2$$

3. Consider the set S defined by

$$z\bar{z} + 2iz - 2i\bar{z} + 1 = 0$$

Is it open/closed/both/neither? Draw a picture. Give the equation of an open disk with centre $2i$ not containing any points of S .

4. Consider the function

$$f(z) = x^2 - x + y + i(y^2 - 5y - x)$$

Determine the points at which it is

- 1) \mathbb{C} -differentiable
- 2) analytic

5. Can there be $z \in \mathbb{C}$ such that $e^z = -1$? If no, why? If yes, find all such z .