Suggested problems for Exam 2

All closed curves should be considered positively oriented, if not stated otherwise.

1 Core Problems with some extras

- **17.6**(from last week)
  25,31,36,41,43,47
- **17.7**
  1,9,13,15,19,21,29
- **17.8**
  3,9,13
- **18.1**
  3,5,13,19,23,25,26,27,28,33
- **18.2**
  1,3,7,9,13,19,21,23
- **18.3**
  1,2,3,4,9,13,19,23

2 Evaluating Line Integrals Directly

Compute the following integrals *directly*, i.e., without using Cauchý 's Theorem.

1) This problem is **very important**!
Let C be the circle $|z - z_0| = a$, and let n be an integer - positive, negative or zero. Show by direct computation that

$$\int_C \frac{1}{(z-z_0)^n} dz = \begin{cases} 0, & n \neq 1 \\ 2\pi i, & n = -1 \end{cases}$$

2) $\int_C (z-1)dz$, where C is :
a) $z-1 = e^{i\theta}, 0 \leq \theta \leq \pi$ b) The segment $[0,2]$.

3) $\int_C Rez\ dz$, where C is :
a) The unit circle, traversed once in positive direction.
b)The segment $[z_1, z_2]$.

4) $\int_C \frac{z+2}{z} dz$ where C is the curve:
a)$z = 2e^{i\theta}, \theta \in [0, \pi]$
b)$z = 2e^{i\theta}, \theta \in [-\pi, \pi]$

5) $\int_C e^z dz$, where C is the segment $[\pi i, 1]$.

6) $\int_C z\cos(z)dz$, where C is the segment $[0, i]$.

7) $\int_C \frac{dz}{\sqrt{z}}$ along the upper half of the circle $|z| = 1$ (in positive direction).

8) Let C be the arc of $|z| = 2$ contained in 1-st quadrant. Show (without computing the integral) that

$$\left| \int_C \frac{dz}{z^2+1} \right| \leq \frac{\pi}{3}$$

9) Show that

$$\left| \int_C (e^z - \bar{z})dz \right| \leq 60$$

where C is the triangle with vertices $-4, 0, 3i$.

3 Some Applications of Cauchý's theorem

1. Compute

$$\int_C \frac{dz}{z^2+4}$$

where C is a simple closed curve, not passing through $\pm 2i$.
Hint: You have to consider different cases.

2. Let C be the circle $|z| = 3$, traversed in positive direction. Compute the integrals

$$\int_C \frac{z(z-2)}{z^3+1} dz, \ \int_C \frac{z^2-1}{z} dz$$

Next week we are going to see some more convenient methods which will also allow us to compute more complicated integrals.

3. This is a continuation of the **very important** problem from the previous section.
   Let C be any simple closed curve, not passing through $a \in \mathbb{C}$, and let n be an integer. Show that

$$\int_C \frac{1}{(z-a)^n} dz = \begin{cases} 0, & n \neq 1 \\ 2\pi i, & n = 1, a \text{ inside } C \\ 0 & n = 1, a \ \text{ outside } C \end{cases}$$