MATH 104 – Practice Midterm Exam 2

The exam will look something like this (in terms of length, type of problems etc):

1. Calculate: $\int_1^4 \sqrt{t}\ln t\,dt$.
2. Calculate the volume obtained by revolving the region between the graph of $y = \dfrac{1}{\sqrt{x^3+x}}$ and the x-axis for $1 \leq x \leq 3$ around the x-axis.
3. The arc of the parabola $y = \sqrt{2x}$ from (2,2) to (8,4) is rotated around the x-axis. Find the surface area of the resulting surface.
4. Calculate $\int e^{\sqrt{x}}\,dx$.
5. Calculate $\int \dfrac{(2\sin x - 3)\cos x}{\sin^2 x - 3\sin x + 2}\,dx$.
6. Solve the initial-value problem $\dfrac{dy}{dx} = x(4+y^2)$, $y(0) = 0$.
7. In a second-order chemical reaction, the reactant A is used up in such a way that the amount of it present decreases at a rate proportional to the square of the amount present. Suppose this reaction begins with 50 grams of A present, and after 10 seconds there are only 25 grams left. How long after the beginning of the reaction will there be only 10 grams left? Will all of the A disappear in a finite time, or will there always be a little bit present?
8. Calculate $\lim_{x\to 0} \dfrac{1-\cos 3x}{x^2}$.
9. Calculate $\lim_{x\to\infty} (e^x + x)^{1/x}$.
10. In the following diagram, x is the horizontal distance along line AB starting from A. Express θ as a function of x. What value of x maximizes θ?

Here are a few more problems from old midterms:

1. Find the arc length of the portion of the graph of the function $f(x) = 2\sec x$ that lies above the interval $[0, \frac{\pi}{4}]$.

2. Calculate $\displaystyle\int_e^{e^4} \frac{1}{x\sqrt{\ln x}}\,dx$

3. Calculate $\displaystyle\int_0^{\sqrt{\ln 2}} \frac{x}{1+e^{x^2}}\,dx$

4. Calculate $\displaystyle\int x \log_8 x^2\,dx$

5. Calculate $\displaystyle\lim_{x\to 0} \frac{3x - \sin(3x)}{x^3}$

6. Calculate $\displaystyle\lim_{x\to\infty} (1+x)^{\frac{1}{3x}}$

7. Solve the initial-value problem:
$$(x+1)\frac{dy}{dx} + 2y = x, \qquad y(0) = 1$$

8. A scientist collects data that relate two variables, x and y. Instead of plotting y as a function of x, she plots $\log_2 y$ as a function of $\log_2 x$, and gets a line whose slope is 3 and whose intercept on the vertical axis is 5. What equation describes y as a function of x?

9. According to Newton's law of heating and cooling, if the temperature of an object is different from the temperature of its environment, then the temperature of the object will change so that the difference between the object's temperature and the ambient temperature decreases at a rate proportional to this difference.

   On a hot day, a thermometer was brought outdoors from an air-conditioned building. The temperature inside the building was 21° C, and so this is what the thermometer read at the moment it was brought outside. One minute later the thermometer read 27° C, and a minute after that it read 31° C. What was the temperature outside? (Impress us and express the answer without using logarithms or the number e.)

10. Suppose $f(x) \geq 0$ for all x, and when the part of the graph of the function $y = f(x)$ between $x = 0$ and $x = a$ is rotated around the x-axis, the solid of revolution so obtained has volume $V(a) = \pi a^2$. Let $S(a)$ be the surface area of that surface. What is $S(a)$? (*Hint*: Start by differentiating the equation $V(a) = \pi a^2$ with respect to a.)