

MATH 104 – Practice Problems for Exam 3

There are too many problems here for one exam, but they're good practice!

For each of the following series, say whether it converges or diverges, and explain why.

1. $\sum_{n=1}^{\infty} \frac{n^3}{n^5 + 3}$

2. $\sum_{n=1}^{\infty} \frac{3^n}{4^n + n^4}$

3. $\sum_{n=1}^{\infty} \frac{n}{2^n}$

4. $\sum_{n=1}^{\infty} n \sin \frac{1}{n}$

5. How many terms of the series $\sum_{n=1}^{\infty} \frac{1}{n^3}$ are needed to get a partial sum that is within 0.001 of the actual sum of the series? Give the smallest possible number of terms.

6. Does $\sum_{n=1}^{\infty} \frac{2^n + 3^n}{4^n}$ converge? If so, what is the sum?

7. For what values of p does the series $\sum_{n=1}^{\infty} \frac{n^p}{2 + n^3}$ converge?

8. Find the precise interval of convergence of the series

$$\sum_{n=1}^{\infty} \frac{(2x - 5)^n}{n^2 4^n}.$$

9. Which of the following is the beginning of the Maclaurin series for $\arctan(x^2)$?

(A) $x^2 - \frac{x^4}{3} + \frac{x^6}{5} - \frac{x^8}{7} + \dots$

(B) $\frac{x^2}{3} - \frac{x^6}{6} + \frac{x^{10}}{9} - \frac{x^{14}}{12} + \dots$

(C) $x^2 - 2x^4 + 3x^6 - 4x^8 + \dots$

(D) $1 + x^4 + 3x^8 + 4x^{12} + \dots$

(E) $\frac{x^2}{3} + \frac{x^6}{6} + \frac{x^{10}}{9} + \frac{x^{14}}{12} + \dots$

(F) $x^2 - \frac{x^6}{3} + \frac{x^{10}}{5} - \frac{x^{14}}{7} + \dots$

10. Let $F(x) = \int_0^x \cos \sqrt{t} dt$. Which of the following is the beginning of the Maclaurin series for F ?

(A) $1 - \frac{x}{2} + \frac{x^2}{24} - \frac{x^3}{720} + \dots$

(B) $x - \frac{x^2}{4} + \frac{x^3}{72} - \frac{x^4}{2880} + \dots$

(C) $x - \frac{x^2}{2} + \frac{x^4}{6} - \frac{x^6}{24} + \dots$

(D) $1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} + \dots$

(E) $x - \frac{x^2}{3} + \frac{x^4}{15} - \frac{x^5}{105} + \dots$

(F) $x + \frac{x^2}{3} + \frac{x^4}{15} + \frac{x^5}{105} + \dots$

11. Use the first two non-zero terms of an appropriate series to give an approximation of

$$\int_0^1 \sin(x^2) dx.$$

Give (with explanation) an estimate of the error (difference between your approximation and the actual value of the integral).

12. We would like to estimate the sum of the series $\sum_{n=1}^{\infty} \frac{1}{n^4 + 3}$, by using the sum of the first ten terms. Of course, the exact error is the sum of all the terms from the 11th on, i.e., $\sum_{n=11}^{\infty} \frac{1}{n^4 + 3}$. Show that this error is less than $1/3000$ by comparing this with the sum of $1/n^4$ and then by estimating this latter sum using an appropriate integral.

13. Find the center and radius of convergence of the power series

$$\sum_{n=2}^{\infty} \frac{(-1)^n \sqrt{1+n}}{n^2} (x-2)^n.$$

(In other words, find the largest *open* interval on which the series converges).

14. Write the second-degree Taylor polynomial for $f(x) = \sqrt{x}$ centered at $a = 25$. Use this polynomial from to estimate $\sqrt{26}$. Also, give an estimate of the error.

15. Determine whether the series $\sum_{n=1}^{\infty} \frac{\sqrt[n]{n}}{n^2}$ converges or diverges. Please explain carefully.

16. Does the sequence $\{(n^2 + n)^{1/n}\}$ converge or diverge? If it converges, calculate its limit.

17. Let $a_n = \int_n^\infty \frac{1}{1 + (x^2 + x^4)^2} dx$. Does the sequence $\{a_n\}$ converge? If so, what is the limit?

18. Determine whether the following series converges:

$$\sum_{n=3}^{\infty} \ln\left(1 + \frac{1}{n^2}\right)$$

19. Does the series $\sum_{n=1}^{\infty} \frac{n!(n+1)!}{(3n)!}$ converge or diverge? Be sure to justify your answer.

20. Find the limit.

$$\lim_{x \rightarrow 0} \frac{\cos x - 1}{e^{x^2} - 1}$$

The rest of these are problems from exams the last few times I've given the course.

21. What is the limit of the sequence $\left\{ \arctan\left(\frac{n^2}{1+n^2}\right) \right\}$?

(a) 0 (b) 1 (c) $\frac{\pi}{4}$ (d) $\ln 3$ (e) $\ln \frac{1}{9}$ (f) diverges

22. Does the series $\sum_{n=2}^{\infty} \frac{1}{n^2 - \sqrt{n}}$ converge or diverge? (Why?)

(a) Converges (b) Diverges

23. What is the limit of the sequence $\left\{ \cos\left(\frac{\sqrt{n}}{1+n}\right) \right\}$?

(a) 0 (b) 1 (c) $\frac{\pi}{4}$ (d) $\ln 3$ (e) $\ln \frac{1}{9}$ (f) diverges

24. Does the series $\sum_{n=2}^{\infty} \frac{1}{n - \sqrt{n}}$ converge or diverge? (Why?)

(a) Converges (b) Diverges

25. Does the series $\sum_{n=1}^{\infty} \frac{n!}{n^5}$ converge or diverge? (Why?)
- (a) Converges (b) Diverges
26. What is the limit of the sequence $\left\{n^2 \left(1 - \cos \frac{1}{n}\right)\right\}$?
- (a) 1 (b) -1 (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{2}$ (e) $-\frac{\sqrt{3}}{2}$ (f) diverges
27. Does the series $\sum_{n=1}^{\infty} \frac{1}{n + \sqrt{n}}$ converge or diverge? (Why?)
- (a) Converges (b) Diverges
28. Does the series $\sum_{n=1}^{\infty} \frac{1}{n^n}$ converge or diverge? (Why?)
- (a) Converges (b) Diverges
29. Does the series $\sum_{n=1}^{\infty} \frac{3^n}{n^3}$ converge or diverge? (Why?)
- (a) Converges (b) Diverges
30. Does the series $\sum_{n=0}^{\infty} \frac{2n + 3}{(n^2 + 3n + 6)^2}$ converge or diverge? (Why?)
- (a) Converges (b) Diverges
31. For which values of x does the series $\sum_{n=0}^{\infty} \frac{(x - 4)^n}{5^n}$ converge? What is the sum?
- (a) $-1 < x < 1, \frac{5}{x - 4}$ (b) $-1 < x < 1, \frac{5}{9 - x}$ (c) $-1 < x < 9, \frac{5}{x - 4}$
 (d) $-1 < x < 9, \frac{5}{9 - x}$ (e) $-1 < x < 1, \frac{4}{5 - x}$ (f) $-1 < x < 9, \frac{4}{5 - x}$
32. Does the series $\sum_{n=1}^{\infty} \frac{(-1)^n 2^n}{3^n + 4^n}$ converge absolutely, converge conditionally, or diverge? (Why?)
- (a) Converges absolutely (b) Converges conditionally (c) Diverges

33. Does the series $\sum_{n=1}^{\infty} \frac{n^5}{n!}$ converge or diverge? (Why?)
- (a) Converges (b) Diverges
34. Does the series $\sum_{n=1}^{\infty} \frac{(-1)^n(n^2 + n^3)}{n^4 + 1}$ converge absolutely, converge conditionally, or diverge? (Why?)
- (a) Converges absolutely (b) Converges conditionally (c) Diverges
35. Does the series $\sum_{n=1}^{\infty} \frac{(2n)!}{2^n(n!)^2}$ converge or diverge? (Why?)
- (a) Converges (b) Diverges
36. Does the series $\sum_{n=1}^{\infty} \frac{2^n(n!)^2}{(2n)!}$ converge or diverge? (Why?)
- (a) Converges (b) Diverges
37. Does the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n \ln n}$ converge absolutely, converge conditionally, or diverge? (Why?)
- (a) Converges absolutely (b) Converges conditionally (c) Diverges
38. For which values of p does the series $\sum_{n=0}^{\infty} \frac{1}{(n^2 + 1)^p}$ converge?
- (a) $p > 1$ (b) $p < -1$ (c) $p > \frac{1}{2}$ (d) $p < \ln 5 - \ln 3$ (e) $p < \frac{\ln 5}{\ln 3}$ (f) $p < 0$
39. For which values of x does the series $\sum_{n=1}^{\infty} \frac{(-1)^n(x - 2)^n}{n3^n}$ converge?
- (a) $-1 < x \leq 5$ (b) $-1 < x < 5$ (c) $-1 \leq x \leq 5$
 (d) $5/3 \leq x \leq 7/3$ (e) $-3 < x < 3$ (f) $-3 \leq x \leq 3$

40. For which values of p does the series $\sum_{n=0}^{\infty} \ln(1 + n^p)$ converge?
- (a) $p > 1$ (b) $p < -1$ (c) $p > \frac{1}{2}$ (d) $p < \ln 5 - \ln 3$ (e) $p < \frac{\ln 5}{\ln 3}$ (f) $p < 0$
41. For which values of x does the series $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n^2 3^n}$ converge?
- (a) $-1 < x \leq 5$ (b) $-1 < x < 5$ (c) $-1 \leq x \leq 5$
(d) $5/3 \leq x \leq 7/3$ (e) $-3 < x < 3$ (f) $-3 \leq x \leq 3$
42. For which values of p does the series $\sum_{n=0}^{\infty} \frac{3^{np}}{4^n + 5^n}$ converge?
- (a) $p > 1$ (b) $p < -1$ (c) $p > \frac{1}{2}$ (d) $p < \ln 5 - \ln 3$ (e) $p < \frac{\ln 5}{\ln 3}$ (f) $p < 0$
43. For which values of x does the series $\sum_{n=1}^{\infty} \frac{3^n(x-2)^n}{n^2}$ converge?
- (a) $-1 < x \leq 5$ (b) $-1 < x < 5$ (c) $-1 \leq x \leq 5$
(d) $5/3 \leq x \leq 7/3$ (e) $-3 < x < 3$ (f) $-3 \leq x \leq 3$
44. What are the first few nonzero terms of the Maclaurin series for $f(x) = xe^{3x}$?
- (a) $1 - 2x^2 + \frac{2}{3}x^4 - \frac{4}{45}x^6 + \dots$ (b) $x + \frac{1}{4}x^2 + \frac{1}{8}x^3 + \frac{1}{12}x^4 + \dots$
(c) $x^2 + \frac{1}{4}x^3 + \frac{1}{16}x^4 + \frac{1}{64}x^5 + \dots$ (d) $1 + 2x^2 + \frac{4}{3}x^4 + \frac{4}{45}x^6 + \dots$
(e) $x^2 - \frac{1}{4}x^3 + \frac{1}{16}x^4 - \frac{1}{64}x^5 + \dots$ (f) $x + 3x^2 + \frac{9}{2}x^3 + \frac{9}{2}x^4 + \dots$
45. Which of the following gives the value of $\int_0^{1/2} e^{-x^5} dx$ correct to within 0.0001 (i.e., within 1/10000)?
- (a) $\frac{1}{2}$ (b) $\frac{1}{2} - \frac{1}{120}$ (c) $\frac{1}{24} - \frac{1}{2688}$ (d) $\frac{1}{2} - \frac{1}{320}$ (e) $\frac{1}{2} - \frac{1}{384}$ (f) $\frac{1}{24} - \frac{1}{1024}$
46. What are the first few nonzero terms of the Maclaurin series for $f(x) = \frac{4x^2}{4-x}$?
- (a) $1 - 2x^2 + \frac{2}{3}x^4 - \frac{4}{45}x^6 + \dots$ (b) $x + \frac{1}{4}x^2 + \frac{1}{8}x^3 + \frac{1}{12}x^4 + \dots$

$$(c) x^2 + \frac{1}{4}x^3 + \frac{1}{16}x^4 + \frac{1}{64}x^5 + \dots$$

$$(d) 1 + 2x^2 + \frac{4}{3}x^4 + \frac{4}{45}x^6 + \dots$$

$$(e) x^2 - \frac{1}{4}x^3 + \frac{1}{16}x^4 - \frac{1}{64}x^5 + \dots$$

$$(f) x + 3x^2 + \frac{9}{2}x^3 + \frac{9}{2}x^4 + \dots$$

47. Which of the following gives the value of $\int_0^{1/2} \cos(x^2) dx$ correct to within 0.0001 (i.e., within 1/10000)?

$$(a) \frac{1}{2} \quad (b) \frac{1}{2} - \frac{1}{120} \quad (c) \frac{1}{24} - \frac{1}{2688} \quad (d) \frac{1}{2} - \frac{1}{320} \quad (e) \frac{1}{2} - \frac{1}{384} \quad (f) \frac{1}{24} - \frac{1}{1024}$$

48. What are the first few nonzero terms of the Maclaurin series for $f(x) = \cos(2x)$?

$$(a) 1 - 2x^2 + \frac{2}{3}x^4 - \frac{4}{45}x^6 + \dots$$

$$(b) x + \frac{1}{4}x^2 + \frac{1}{8}x^3 + \frac{1}{12}x^4 + \dots$$

$$(c) x^2 + \frac{1}{4}x^3 + \frac{1}{16}x^4 + \frac{1}{64}x^5 + \dots$$

$$(d) 1 + 2x^2 + \frac{4}{3}x^4 + \frac{4}{45}x^6 + \dots$$

$$(e) x^2 - \frac{1}{4}x^3 + \frac{1}{16}x^4 - \frac{1}{64}x^5 + \dots$$

$$(f) x + 3x^2 + \frac{9}{2}x^3 + \frac{9}{2}x^4 + \dots$$

49. Which of the following gives the value of $\int_0^{1/2} \arctan(x^2) dx$ correct to within 0.0001 (i.e., within 1/10000)?

$$(a) \frac{1}{2} \quad (b) \frac{1}{2} - \frac{1}{120} \quad (c) \frac{1}{24} - \frac{1}{2688} \quad (d) \frac{1}{2} - \frac{1}{320} \quad (e) \frac{1}{2} - \frac{1}{384} \quad (f) \frac{1}{24} - \frac{1}{1024}$$

MATH 104-004 – Third Midterm Exam - Fall 2010

1. What is the limit of the sequence $\{\sin(\arctan(\ln(n)))\}$?

1. What is the limit of the sequence $\{\cos(\arctan(\ln(n)))\}$?

1. What is the limit of the sequence $\{\ln(\cos(\arctan(n)))\}$?

$$(a) 0 \quad (b) 1 \quad (c) \frac{\pi}{4} \quad (d) \ln 2 \quad (e) \frac{\pi}{2} \quad (f) \text{diverges}$$

2. Does the series $\sum_{n=2}^{\infty} \frac{n}{n^3 - \sqrt{n} + 1}$ converge or diverge? (Why?)

2. Does the series $\sum_{n=2}^{\infty} \frac{n^2}{n^3 - \sqrt{n} + 1}$ converge or diverge? (Why?)

2. Does the series $\sum_{n=2}^{\infty} \frac{\sqrt{n}}{n^2 - n + 1}$ converge or diverge? (Why?)

(a) Converges

(b) Diverges

3. Does the series $\sum_{n=1}^{\infty} \frac{2^n}{n^5}$ converge or diverge? (Why?)

3. Does the series $\sum_{n=1}^{\infty} \frac{n^5}{2^n}$ converge or diverge? (Why?)

3. Does the series $\sum_{n=1}^{\infty} \frac{n^5 5^n}{n!}$ converge or diverge? (Why?)

(a) Converges

(b) Diverges

4. Does the series $\sum_{n=1}^{\infty} \frac{(-1)^n (n^2 + n)}{n^3 + 2n + 1}$ converge absolutely, converge conditionally, or diverge? (Why?)

4. Does the series $\sum_{n=1}^{\infty} \frac{(-1)^n (n^2 + n^3)}{n^5 + 1}$ converge absolutely, converge conditionally, or diverge? (Why?)

4. Does the series $\sum_{n=1}^{\infty} \frac{(-1)^n (n^2 + n^3)}{n^4 + \ln n}$ converge absolutely, converge conditionally, or diverge? (Why?)

(a) Converges absolutely

(b) Converges conditionally

(c) Diverges

5. If it converges, find the sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n2^n}$. If the series diverges, explain why.

5. If it converges, find the sum of the series $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{9^n (2n)!}$. If the series diverges, explain why.

5. If it converges, find the sum of the series $\sum_{n=0}^{\infty} \frac{(-1)^n}{n! 2^n}$. If the series diverges, explain why.

(a) $\ln 2$ (b) $\ln 3 - \ln 2$ (c) $1/\sqrt{e}$ (d) $1/2$ (e) $1/e^2$ (f) Diverges

6. Does the series $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln(n!)}$ converge absolutely, converge conditionally, or diverge? (Why?)

6. Does the series $\sum_{n=2}^{\infty} \frac{(-1)^n \sqrt{n^3 + 1}}{\ln(n!)}$ converge absolutely, converge conditionally, or diverge? (Why?)

6. Does the series $\sum_{n=2}^{\infty} \frac{(-1)^n \ln(n!)}{\sqrt{n^5 + 1}}$ converge absolutely, converge conditionally, or diverge? (Why?)

(a) Converges absolutely (b) Converges conditionally (c) Diverges

7. For which values of p does the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{1 + n^p}}$ converge?

7. For which values of p does the series $\sum_{n=1}^{\infty} \sqrt{1 + n^p}$ converge?

7. For which values of p does the series $\sum_{n=1}^{\infty} \frac{n}{1 + n^p}$ converge?

(a) $p > 1$ (b) $p < -1$ (c) $p > \frac{1}{2}$ (d) $p > 2$ (e) $p < 0$ (f) no value of p

8. For which values of x does the series $\sum_{n=1}^{\infty} \frac{(x-1)^n}{n^2}$ converge?

8. For which values of x does the series $\sum_{n=1}^{\infty} \frac{(x-1)^n}{1 + 2^n}$ converge?

8. For which values of x does the series $\sum_{n=1}^{\infty} \frac{nx^n}{2^n}$ converge?

(a) $-1 < x < 3$

(b) $0 < x < 2$

(c) $-1 \leq x \leq 3$

(d) $0 \leq x \leq 2$

(e) $-2 < x < 2$

(f) $-2 \leq x \leq 2$

9. What are the first few nonzero terms of the Maclaurin series for $f(x) = x \cos(2x)$?

9. What are the first few nonzero terms of the Maclaurin series for $f(x) = \frac{1}{2} \sin(2x)$?

9. What are the first few nonzero terms of the Maclaurin series for $f(x) = xe^{-2x^2}$?

(a) $1 - 2x^2 + \frac{2}{3}x^4 - \frac{4}{45}x^6 + \dots$

(b) $x + \frac{1}{4}x^2 + \frac{1}{8}x^3 + \frac{1}{12}x^4 + \dots$

(c) $x^2 + \frac{1}{4}x^3 + \frac{1}{16}x^4 + \frac{1}{64}x^5 + \dots$

(d) $x - 2x^3 + \frac{2}{3}x^5 - \frac{4}{45}x^7 + \dots$

(e) $x - \frac{2}{3}x^3 + \frac{2}{15}x^5 - \frac{4}{315}x^7 + \dots$

(f) $x - 2x^3 + 2x^5 - \frac{4}{3}x^7 + \dots$

10. Which of the following is closest to the value of $\int_0^{0.1} \sqrt{1+x^2} dx$? Justify your answer.

10. Which of the following is closest to the value of $\int_0^{0.1} e^{-x^2/2} dx$? Justify your answer.

10. Which of the following is closest to the value of $\int_0^{0.1} \cos(x^2) dx$? Justify your answer.

(a) 0.0998

(b) 0.0999

(c) 0.1000

(d) 0.1001

(e) 0.1002

(f) 0.1003