MATH 104 – Practice Problems for Exam 3

There are too many problems here for one exam, but they're good practice!

For each of the following series, say whether it converges or diverges, and explain why.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{n^3}{n^5+3}$

2. $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{4^n+n^4}$

3. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{2^n}$

4. $\displaystyle\sum_{n=1}^{\infty} n \sin\frac{1}{n}$

5. How many terms of the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^3}$ are needed to get a partial sum that is within 0.001 of the actual sum of the series? Give the smallest possible number of terms.

6. Does $\displaystyle\sum_{n=1}^{\infty} \frac{2^n+3^n}{4^n}$ converge? If so, what is the sum?

7. For what values of p does the series $\displaystyle\sum_{n=1}^{\infty} \frac{n^p}{2+n^3}$ converge?

8. Find the precise interval of convergence of the series

$$\sum_{n=1}^{\infty} \frac{(2x-5)^n}{n^2 4^n}.$$

9. Which of the following is the beginning of the Maclaurin series for $\arctan(x^2)$?

(A) $x^2 - \dfrac{x^4}{3} + \dfrac{x^6}{5} - \dfrac{x^8}{7} + \ldots$

(B) $\dfrac{x^2}{3} - \dfrac{x^6}{6} + \dfrac{x^{10}}{9} - \dfrac{x^{14}}{12} + \ldots$

(C) $x^2 - 2x^4 + 3x^6 - 4x^8 + \ldots$

(D) $1 + x^4 + 3x^8 + 4x^{12} + \ldots$

(E) $\dfrac{x^2}{3} + \dfrac{x^6}{6} + \dfrac{x^{10}}{9} + \dfrac{x^{14}}{12} + \ldots$

(F) $x^2 - \dfrac{x^6}{3} + \dfrac{x^{10}}{5} - \dfrac{x^{14}}{7} + \ldots$

10. Let $F(x) = \int_0^x \cos\sqrt{t}\,dt$. Which of the following is the beginning of the Maclaurin series for F?

(A) $1 - \frac{x}{2} + \frac{x^2}{24} - \frac{x^3}{720} + \dots$

(B) $x - \frac{x^2}{4} + \frac{x^3}{72} - \frac{x^4}{2880} + \dots$

(C) $x - \frac{x^2}{2} + \frac{x^4}{6} - \frac{x^6}{24} + \dots$

(D) $1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} + \dots$

(E) $x - \frac{x^2}{3} + \frac{x^4}{15} - \frac{x^5}{105} + \dots$

(F) $x + \frac{x^2}{3} + \frac{x^4}{15} + \frac{x^5}{105} + \dots$

11. Use the first two non-zero terms of an appropriate series to give an approximation of

$$\int_0^1 \sin(x^2)\,dx.$$

Give (with explanation) an estimate of the error (difference between your approximation and the actual value of the integral).

12. We would like to estimate the sum of the series $\sum_{n=1}^{\infty} \frac{1}{n^4+3}$, by using the sum of the first ten terms. Of course, the exact error is the sum of all the terms from the 11th on, i.e., $\sum_{n=11}^{\infty} \frac{1}{n^4+3}$. Show that this error is less than $1/3000$ by comparing this with the sum of $1/n^4$ and then by estimating this latter sum using an appropriate integral.

13. Find the center and radius of convergence of the power series

$$\sum_{n=2}^{\infty} \frac{(-1)^n\sqrt{1+n}}{n^2}(x-2)^n.$$

(In other words, find the largest *open* interval on which the series converges).

14. Write the second-degree Taylor polynomial for $f(x) = \sqrt{x}$ centered at $a = 25$. Use this polynomial from to estimate $\sqrt{26}$. Also, give an estimate of the error.

15. Determine whether the series $\sum_{n=1}^{\infty} \frac{\sqrt[n]{n}}{n^2}$ converges or diverges. Please explain carefully.

16. Does the sequence $\{(n^2+n)^{1/n}\}$ converge or diverge? If it converges, calculate its limit.

17. Let $a_n = \displaystyle\int_n^\infty \frac{1}{1+(x^2+x^4)^2}dx$. Does the sequence $\{a_n\}$ converge? If so, what is the limit?

18. Determine whether the following series converges:

$$\sum_{n=3}^{\infty} \ln(1+\frac{1}{n^2})$$

19. Does the series $\displaystyle\sum_{n=1}^{\infty} \frac{n!(n+1)!}{(3n)!}$ converge or diverge? Be sure to justify your answer.

20. Find the limit.

$$\lim_{x\to 0} \frac{\cos x - 1}{e^{x^2}-1}$$

The rest of these are problems from exams the last few times I've given the course.

21. What is the limit of the sequence $\left\{\arctan\left(\dfrac{n^2}{1+n^2}\right)\right\}$?

(a) 0 (b) 1 (c) $\dfrac{\pi}{4}$ (d) $\ln 3$ (e) $\ln\dfrac{1}{9}$ (f) diverges

22. Does the series $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^2-\sqrt{n}}$ converge or diverge? (Why?)

(a) Converges (b) Diverges

23. What is the limit of the sequence $\left\{\cos\left(\dfrac{\sqrt{n}}{1+n}\right)\right\}$?

(a) 0 (b) 1 (c) $\dfrac{\pi}{4}$ (d) $\ln 3$ (e) $\ln\dfrac{1}{9}$ (f) diverges

24. Does the series $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n-\sqrt{n}}$ converge or diverge? (Why?)

(a) Converges (b) Diverges

25. Does the series $\sum_{n=1}^{\infty} \frac{n!}{n^5}$ converge or diverge? (Why?)

(a) Converges (b) Diverges

26. What is the limit of the sequence $\left\{n^2\left(1-\cos\frac{1}{n}\right)\right\}$?

(a) 1 (b) -1 (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{2}$ (e) $-\frac{\sqrt{3}}{2}$ (f) diverges

27. Does the series $\sum_{n=1}^{\infty} \frac{1}{n+\sqrt{n}}$ converge or diverge? (Why?)

(a) Converges (b) Diverges

28. Does the series $\sum_{n=1}^{\infty} \frac{1}{n^n}$ converge or diverge? (Why?)

(a) Converges (b) Diverges

29. Does the series $\sum_{n=1}^{\infty} \frac{3^n}{n^3}$ converge or diverge? (Why?)

(a) Converges (b) Diverges

30. Does the series $\sum_{n=0}^{\infty} \frac{2n+3}{(n^2+3n+6)^2}$ converge or diverge? (Why?)

(a) Converges (b) Diverges

31. For which values of x does the series $\sum_{n=0}^{\infty} \frac{(x-4)^n}{5^n}$ converge? What is the sum?

(a) $-1 < x < 1$, $\frac{5}{x-4}$ (b) $-1 < x < 1$, $\frac{5}{9-x}$ (c) $-1 < x < 9$, $\frac{5}{x-4}$

(d) $-1 < x < 9$, $\frac{5}{9-x}$ (e) $-1 < x < 1$, $\frac{4}{5-x}$ (f) $-1 < x < 9$, $\frac{4}{5-x}$

32. Does the series $\sum_{n=1}^{\infty} \frac{(-1)^n 2^n}{3^n+4^n}$ converge absolutely, converge conditionally, or diverge? (Why?)

(a) Converges absolutely (b) Converges conditionally (c) Diverges

33. Does the series $\sum_{n=1}^{\infty} \frac{n^5}{n!}$ converge or diverge? (Why?)

(a) Converges (b) Diverges

34. Does the series $\sum_{n=1}^{\infty} \frac{(-1)^n(n^2+n^3)}{n^4+1}$ converge absolutely, converge conditionally, or diverge? (Why?)

(a) Converges absolutely (b) Converges conditionally (c) Diverges

35. Does the series $\sum_{n=1}^{\infty} \frac{(2n)!}{2^n(n!)^2}$ converge or diverge? (Why?)

(a) Converges (b) Diverges

36. Does the series $\sum_{n=1}^{\infty} \frac{2^n(n!)^2}{(2n)!}$ converge or diverge? (Why?)

(a) Converges (b) Diverges

37. Does the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n \ln n}$ converge absolutely, converge conditionally, or diverge? (Why?)

(a) Converges absolutely (b) Converges conditionally (c) Diverges

38. For which values of p does the series $\sum_{n=0}^{\infty} \frac{1}{(n^2+1)^p}$ converge?

(a) $p > 1$ (b) $p < -1$ (c) $p > \frac{1}{2}$ (d) $p < \ln 5 - \ln 3$ (e) $p < \frac{\ln 5}{\ln 3}$ (f) $p < 0$

39. For which values of x does the series $\sum_{n=1}^{\infty} \frac{(-1)^n(x-2)^n}{n3^n}$ converge?

(a) $-1 < x \leq 5$ (b) $-1 < x < 5$ (c) $-1 \leq x \leq 5$

(d) $5/3 \leq x \leq 7/3$ (e) $-3 < x < 3$ (f) $-3 \leq x \leq 3$

40. For which values of p does the series $\sum_{n=0}^{\infty} \ln(1+n^p)$ converge?

(a) $p > 1$ (b) $p < -1$ (c) $p > \frac{1}{2}$ (d) $p < \ln 5 - \ln 3$ (e) $p < \dfrac{\ln 5}{\ln 3}$ (f) $p < 0$

41. For which values of x does the series $\sum_{n=1}^{\infty} \dfrac{(x-2)^n}{n^2 3^n}$ converge?

(a) $-1 < x \leq 5$ (b) $-1 < x < 5$ (c) $-1 \leq x \leq 5$
(d) $5/3 \leq x \leq 7/3$ (e) $-3 < x < 3$ (f) $-3 \leq x \leq 3$

42. For which values of p does the series $\sum_{n=0}^{\infty} \dfrac{3^{np}}{4^n + 5^n}$ converge?

(a) $p > 1$ (b) $p < -1$ (c) $p > \frac{1}{2}$ (d) $p < \ln 5 - \ln 3$ (e) $p < \dfrac{\ln 5}{\ln 3}$ (f) $p < 0$

43. For which values of x does the series $\sum_{n=1}^{\infty} \dfrac{3^n (x-2)^n}{n^2}$ converge?

(a) $-1 < x \leq 5$ (b) $-1 < x < 5$ (c) $-1 \leq x \leq 5$
(d) $5/3 \leq x \leq 7/3$ (e) $-3 < x < 3$ (f) $-3 \leq x \leq 3$

44. What are the first few nonzero terms of the Maclaurin series for $f(x) = xe^{3x}$?

(a) $1 - 2x^2 + \dfrac{2}{3}x^4 - \dfrac{4}{45}x^6 + \cdots$ (b) $x + \dfrac{1}{4}x^2 + \dfrac{1}{8}x^3 + \dfrac{1}{12}x^4 + \cdots$
(c) $x^2 + \dfrac{1}{4}x^3 + \dfrac{1}{16}x^4 + \dfrac{1}{64}x^5 + \cdots$ (d) $1 + 2x^2 + \dfrac{4}{3}x^4 + \dfrac{4}{45}x^6 + \cdots$
(e) $x^2 - \dfrac{1}{4}x^3 + \dfrac{1}{16}x^4 - \dfrac{1}{64}x^5 + \cdots$ (f) $x + 3x^2 + \dfrac{9}{2}x^3 + \dfrac{9}{2}x^4 + \cdots$

45. Which of the following gives the value of $\int_0^{1/2} e^{-x^5}\,dx$ correct to within 0.0001 (i.e., within 1/10000)?

(a) $\dfrac{1}{2}$ (b) $\dfrac{1}{2} - \dfrac{1}{120}$ (c) $\dfrac{1}{24} - \dfrac{1}{2688}$ (d) $\dfrac{1}{2} - \dfrac{1}{320}$ (e) $\dfrac{1}{2} - \dfrac{1}{384}$ (f) $\dfrac{1}{24} - \dfrac{1}{1024}$

46. What are the first few nonzero terms of the Maclaurin series for $f(x) = \dfrac{4x^2}{4-x}$?

(a) $1 - 2x^2 + \dfrac{2}{3}x^4 - \dfrac{4}{45}x^6 + \cdots$ (b) $x + \dfrac{1}{4}x^2 + \dfrac{1}{8}x^3 + \dfrac{1}{12}x^4 + \cdots$

(c) $x^2 + \frac{1}{4}x^3 + \frac{1}{16}x^4 + \frac{1}{64}x^5 + \cdots$

(d) $1 + 2x^2 + \frac{4}{3}x^4 + \frac{4}{45}x^6 + \cdots$

(e) $x^2 - \frac{1}{4}x^3 + \frac{1}{16}x^4 - \frac{1}{64}x^5 + \cdots$

(f) $x + 3x^2 + \frac{9}{2}x^3 + \frac{9}{2}x^4 + \cdots$

47. Which of the following gives the value of $\int_0^{1/2} \cos(x^2)\,dx$ correct to within 0.0001 (i.e., within 1/10000)?

(a) $\frac{1}{2}$ (b) $\frac{1}{2} - \frac{1}{120}$ (c) $\frac{1}{24} - \frac{1}{2688}$ (d) $\frac{1}{2} - \frac{1}{320}$ (e) $\frac{1}{2} - \frac{1}{384}$ (f) $\frac{1}{24} - \frac{1}{1024}$

48. What are the first few nonzero terms of the Maclaurin series for $f(x) = \cos(2x)$?

(a) $1 - 2x^2 + \frac{2}{3}x^4 - \frac{4}{45}x^6 + \cdots$

(b) $x + \frac{1}{4}x^2 + \frac{1}{8}x^3 + \frac{1}{12}x^4 + \cdots$

(c) $x^2 + \frac{1}{4}x^3 + \frac{1}{16}x^4 + \frac{1}{64}x^5 + \cdots$

(d) $1 + 2x^2 + \frac{4}{3}x^4 + \frac{4}{45}x^6 + \cdots$

(e) $x^2 - \frac{1}{4}x^3 + \frac{1}{16}x^4 - \frac{1}{64}x^5 + \cdots$

(f) $x + 3x^2 + \frac{9}{2}x^3 + \frac{9}{2}x^4 + \cdots$

49. Which of the following gives the value of $\int_0^{1/2} \arctan(x^2)\,dx$ correct to within 0.0001 (i.e., within 1/10000)?

(a) $\frac{1}{2}$ (b) $\frac{1}{2} - \frac{1}{120}$ (c) $\frac{1}{24} - \frac{1}{2688}$ (d) $\frac{1}{2} - \frac{1}{320}$ (e) $\frac{1}{2} - \frac{1}{384}$ (f) $\frac{1}{24} - \frac{1}{1024}$