**MATH 104 – Practice Problems for Exam 3 - Hints and answers**

1. Converges by comparison (or limit comparison) with $\sum(1/n^2)$, a convergent $p$-series.

2. Converges by comparison (or limit comparison) with $\sum(\frac{3}{4})^n$, a convergent geometric series. You could also use the ratio test.

3. Converges by the ratio test (ratio turns out to be 1/2).

4. Diverges by $n$th term test – $\lim_{n\to\infty} n \sin \frac{1}{n} = 1$.

5. Using the idea of the integral test, see that what's left over after the first $N$ terms, namely $\sum_{n=N+1}^{\infty} \frac{1}{n^3}$ is less than $\int_N^{\infty} \frac{1}{x^3}\,dx$, which equals $\frac{1}{2N^2}$. For this to be less than 0.001, we need $N^2 > 500$, in other words $N \geq 23$. So 23 terms will do.

6. The series given here converges by the ratio test (or by comparison with the convergent $\sum(\frac{3}{4})^n$). This is the sum of two geometric series, and the sum equals 4.

7. Use limit comparison with $\sum 1/n^{3-p}$, which converges when $p < 2$ (which is the answer).

8. Ratio test gives $1/2 < x < 9/2$. Checking endpoints shows that the series converges at both ends (has either 1 or $(-1)^n$ over $n^2$), so the precise interval is $[1/2, 9/2]$.

9. Recall $\arctan(x) = x - \frac{x^3}{3} + \frac{x^5}{5} + \cdots$, so it looks like F.

10. Substitute $\sqrt{t}$ into the series for $\cos x$ (which works because all the powers of $x$ in that series are even) and integrate. I got B.

11. Integrate $x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} + \cdots$ (keep the extra term to estimate the error) and get $\frac{1}{3} - \frac{1}{42} + \frac{1}{1320} - \cdots$. The sum of the first two terms is 13/42, and because the series is alternating and the terms decreasing, the error is less than the first omitted term, namely 1/1320.

12. This is like problem 5, and once you decide (as indicated) to use that the error is less than $\sum_{11}^{\infty} \frac{1}{n^4}$, the integral test idea gives that this is in turn less than $\int_{10}^{\infty} \frac{1}{x^4}\,dx$, which is exactly 1/3000.

13. The ratio test gives convergence on $1 < x < 3$, which has center at $x = 2$ and radius 1.

14. Using Taylor's formula gives $\sqrt{x} = 5 + \frac{x-25}{10} - \frac{(x-25)^2}{100} + \cdots$. This will be an alternating series and is decreasing. For $x = 26$, get $\sqrt{26}$ is about 5.099. The error is less than the next term, which turns out to be 1/50000.

15. You could write the $n$th term as $1/n^{2-1/n}$, which is less than $1/2^{3/2}$ once $n > 2$. So this series converges by comparison to $\sum 1/n^{3/2}$.

16. Use fancy L'Hosiptal (i.e., write the $n$th term as $e$ to something, and take the limit of the something using L'Hospital's rule) and get the sequence converges to 1.

17. This sequence converges to zero because each term is positive but less than $\int_{n}^{\infty} \frac{1}{x^8}\,dx$, which approaches zero as $n \to \infty$.

18. This series converges by limit comparison to $\sum 1/n^2$. (Try it, and use L'Hospital's rule to evaluate the resulting limit)

19. This series converges by the ratio test (limiting ratio turns out to be 0).

20. Use series or L'Hospital several times – limit is $-1/2$.

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| 21. C | 26. D | 31. D | 36. A | 41. C | 46. C |
| 22. A | 27. B | 32. A | 37. B | 42. E | 47. D |
| 23. B | 28. A | 33. A | 38. C | 43. D | 48. A |
| 24. B | 29. B | 34. B | 39. A | 44. F | 49. C |
| 25. B | 30. A | 35. B | 40. B | 45. E | |