MATH 104 – Sample problems for first exam - Fall 2008

1. $\displaystyle\int_e^{e^4} \frac{1}{x\sqrt{\ln x}}\,dx =$

   (a) 0 (b) 2 (c) 4 (d) e^2 (e) $e^4 - e$

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2. What is the volume of the solid obtained by rotating the area between the graph of $y = \dfrac{1}{1+e^{x^2}}$ and the x-axis for $0 \le x \le \sqrt{\ln 2}$ around the y-axis.

   (a) $5\pi \ln 2 - 3\pi \ln 3$ (b) $8\pi \ln 2 - 4\pi \ln 3$ (c) $15\pi \ln 3 - 10\pi \ln 2$

   (d) $2\pi \ln 2 - \pi \ln 3$ (e) $42\pi \ln 2 - 24\pi \ln 3$

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3. $\displaystyle\int_1^e x(\ln x)\,dx =$

   (a) $\dfrac{e^4}{4} - \dfrac{4}{e^4}$ (b) $\dfrac{e^4}{2} - \dfrac{2}{e^4}$ (c) $\dfrac{3e^4}{4}$

   (d) $\dfrac{e^4}{4} + \dfrac{1}{4}$ (e) $\dfrac{3e^4}{2} - \dfrac{1}{4e^4}$ (f) $\dfrac{e^4}{4}$

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4. What is the volume of the solid obtained by rotating the part of the graph of $y = \cos x\sqrt{\sin x}$ between $x = 0$ and $x = \frac{\pi}{2}$ around the x-axis?

   (a) π (b) $\dfrac{\pi}{2}$ (c) $\dfrac{\pi}{3}$

   (d) $\dfrac{\pi}{4}$ (e) $\dfrac{\pi}{6}$

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5. $\displaystyle\lim_{b\to\infty} \int_0^b \frac{\arctan x}{1+x^2}\,dx =$

   (a) $\dfrac{\pi^2}{2}$ (b) π^2 (c) $\dfrac{\pi^2}{4}$ (d) $\dfrac{\pi^2}{8}$ (e) $+\infty$

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6. The area between a curve $y = f(x)$ and the x-axis between $x = 1$ and $x = b$ is rotated around the x axis and the volume of the resulting solid is $\dfrac{\pi}{3}(\ln b)^3$, for all $b > 1$. What is $f(x)$?

   (a) $(\ln x)^2$ (b) $\dfrac{\ln x}{\sqrt{x}}$ (c) $\dfrac{\ln x}{x}$ (d) $\dfrac{(\ln x)^2}{x}$ (e) $\dfrac{(\ln x)^2}{\sqrt{x}}$

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7. $\int_0^{\pi} \sin^2(2x)\,dx =$

(a) 0 (b) 1 (c) 1/2 (d) $\pi/2$ (e) π (f) 2π

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8. The area bounded by the curves $y = x^2 - 1$ and $y = 2x + 7$ is

(a) 4 (b) 9 (c) 16 (d) 24 (e) 30 (f) 36

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9. $\dfrac{d}{dx}\int_0^{\sin x} e^{t^2}\,dt =$

(a) $e^{\sin^2 x}$ (b) $\cos x\, e^{\sin^2 x}$ (c) $2\sin x\, \cos x\, e^{\sin^2 x}$

(d) $\cos x + e^{\sin^2 x}$ (e) $2\sin x\, e^{\sin^2 x} + e^{\sin^2 x}$ (f) $e^{\cos^2 x}$

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10. What is the average value of the function $f(x) = \sqrt{\dfrac{\arcsin x}{1 - x^2}}$ on the interval $[0, \frac{1}{2}]$?

(a) $\dfrac{\pi\sqrt{6}}{15}$ (b) $\dfrac{\pi^{3/2}\sqrt{2}}{54}$ (c) $\dfrac{\pi\sqrt{3}}{18}$ (d) $\dfrac{\pi^{3/2}\sqrt{6}}{27}$ (e) $\dfrac{\pi\sqrt{2}}{48}$

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11. The functions $\cosh x$ and $\sinh x$ are defined as $\cosh x = \frac{1}{2}(e^x + e^{-x})$ and $\sinh x = \frac{1}{2}(e^x - e^{-x})$. Calculate the area bounded by the curves $y = \cosh x$, $y = \sinh x$, $x = 0$ and $x = b$. What is the limit of this area as $b \to \infty$?

(a) 1 (b) e (c) $e + \dfrac{1}{e}$ (d) $\dfrac{1}{e}$ (e) ∞

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12. What is the volume obtained by revolving the region bounded by $y = x^2 - 4$ and $y = 4 - x^2$ around the line $x = 2$?

(a) $\dfrac{16\pi}{3}$ (b) $\dfrac{64\pi}{3}$ (c) $\dfrac{256\pi}{3}$ (d) $\dfrac{512\pi}{3}$ (e) $\dfrac{1024\pi}{3}$

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13. $\int_0^{\pi^2/4} \dfrac{\cos\sqrt{x}}{\sqrt{x}}\,dx =$

(a) 0 (b) 1 (c) $\sqrt{2}$ (d) 2 (e) $2\sqrt{2}$ (f) 3

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14. Compute $\int (\sin^2 x + 8)^7 \sin x \cos x\,dx$.

(a) $\frac{1}{16}(\sin^2 x + 8)^8 + C$ (b) $(\cos^2 x + 8)^7 + C$ (c) $\frac{1}{2}(\sin^2 x + 8)^8 + C$

(d) $\frac{1}{8}(\sin^2(x + 8)^8 + C$ (e) $\frac{1}{8}(\cos^2 x + 8)^8 + C$ (f) $\frac{1}{32}(\sin^2 x + 8)^7 + C$

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15. Suppose f is a continuous function and $\int_1^9 f(x)\,dx = 6$. Then $\int_1^3 xf(x^2)\,dx =$

(a) 6 (b) 4 (c) 3 (d) 2 (e) 1 (f) 0